Exam III: Econ 500 (December 15, Fall 2000)

This is a closed book exam. You may use a calculator. Please write your name on the cover of your answer book, and return this exam page.

(1) (15 pts).
   (i) Describe the concept that two events A and B are independent;
   (ii) Suppose A and B are independent, show that \( \bar{A} \) and B are independent;
   (iii) The 1999-2000 is the last season that the #1 football team in the Big Ten Conference will automatically play in the Rose Bowl. Suppose there are no ties for its #1 and #2 teams. How many outcomes for the #1 and #2 teams are possible in Big Ten conference (5 pts)?

(2) (10 pts) Prove or disprove: \( \text{Cov}(X, X+Y) + \text{Cov}(Y, X+Y) = \text{Var}(X+Y) \).

(3) (10 pts) Find \( E[X_i - \bar{X}] \) and \( \text{Cov}[X_i, \bar{X}] \) in terms of \( \mu \) and \( \sigma^2 \), where \( X_1, \ldots, X_n \) are i.i.d. (identically and independently distributed) random variables with \( \mu \) and \( \sigma^2 \).

(4) (15 pts). Let \( X \sim \chi^2_{n_x} \) and \( Y \sim \chi^2_{n_y} \) be independent, then
\[
\frac{X/n_x}{Y/n_y} = \frac{\chi^2_{n_x} / n_x}{\chi^2_{n_y} / n_y} = F_{n_x, n_y}
\]
follows F-distribution with numerator degree of freedom \( n_x \) and denominator degree of freedom \( n_y \).

Now let \( X_1, \ldots, X_{n_x} \) be a random sample from \( N(\mu_x, \sigma^2_x) \), and \( Y_1, \ldots, Y_{n_y} \) be a random sample from \( N(\mu_y, \sigma^2_y) \). Define \( s^2_x = \frac{\sum_{i=1}^{n_x} (x_i - \bar{X})^2}{n_x - 1} \), \( s^2_y = \frac{\sum_{i=1}^{n_y} (y_i - \bar{Y})^2}{n_y - 1} \). Explain why \( \frac{s^2_x}{\sigma^2_x} / \frac{s^2_y}{\sigma^2_y} \) is \( F_{n_x-1, n_y-1} \).

(5) (20 pts). A survey of 196 students has a sample variance \( s^2_x = 1.4^2 \) and a sample mean \( \bar{X} = 4.1 \) for their satisfaction (on scales of 1, 2, \ldots, 5) with the final Supreme Court ruling on Tuesday (December 12) on 2000 presidential election.
   (i) Find the 90\% confidence interval for the population mean.
   (ii) A Bush supporter claims that the population mean of satisfaction is at least 4.5. Test this claim at the 5\% significance level.

(6) (30 pts). Suppose daily gasoline consumption per household in Ames is normally distributed. A sample of 16 household are selected, resulting in \( \bar{X} = 3.5 \) gallons and \( s^2_x = 0.9^2 \).
   (i) Estimate daily gasoline consumption per household using a 90% confidence interval.
   (ii) A gas station worker claims that daily gasoline consumption per household is no more than 3 gallons. Test this claim at the 5% significance level.
   (iii) Suppose the population variance, in stead of the sample variance, is equal to \( \sigma^2_x = 0.9^2 \). How will this affect the quality of your estimation? Provide an explanation.