

# MEASURING WELFARE CHANGE

## 1. INTRODUCTION

Welfare economics is first and foremost a policy science. In his classic treatise, A.K. Sen [30] says "Welfare economics is concerned with policy recommendations". Gunnar Myrdal [27] in discussing the economist and the politician proposes that "The politician may reasonably expect from the economist that he should explain the actual situation and state the effects of different modes of action in relation to the same initial situation". Empirical welfare economics provides guidance to policy makers in evaluating policies in actual or simulated markets using available data and reasonable methods. Three requirements of proposed measurement methods seem essential.

- 1.: For an individual or homogeneous group, the welfare measure can answer the question proposed by policymakers.
- 2.: The welfare measure can be calculated from empirically estimated parameters that represent individual optimizing behavior.
- 3.: The welfare measure can be aggregated across individuals or homogeneous groups so as to obtain an overall measure of the desirability of a given policy.

## 2. ALTERNATIVE WAYS TO REPRESENT PREFERENCES

**2.1. Utility Function.** Consider a consumption set is  $X = R_+^L = \{x \in R^L : x_\ell \geq 0 \text{ for } \ell = 1, \dots, L\}$ . A function  $v: X \rightarrow R^1$  is a utility function representing the preference relation  $\succeq$  if, for all  $x^1, x^2 \in X$ ,

$$x^1 \succeq x^2 \Leftrightarrow v(x^1) \geq v(x^2) \quad (1)$$

We typically denote the utility function by  $v(x)$ .

**2.2. Indirect Utility Function.** We obtain the indirect utility function by substituting the utility maximizing levels of  $x$  for a given set of prices,  $p$ , and income,  $m$ , into the utility function.

$$\psi(m, p) = v[x_1(m, p), x_2(m, p), \dots, x_n(m, p)] \quad (2)$$

or

$$\psi(m, p) = \max_x [v(x) : px = m] \quad (3)$$

The indirect utility function gives the maximum utility level obtainable with a given set of prices and income ( $p, m$ ). The indirect utility function is an ordinal ranking of price and income combinations.

**2.3. Cost Function.** The cost function gives the minimum cost way of obtaining a particular utility level for a given set of prices.

$$\begin{aligned} c(u, p) &= \min_{x \geq 0} \sum_{i=1}^n p_i x_i \quad s.t. \quad v(x) = u \\ &= \min_x [px : v(x) = u] \end{aligned} \quad (4)$$

**2.4. Money Metric Utility Function.** The money metric utility function gives the minimum cost of obtaining the utility of the vector  $x$  when prices are  $p$ . Specifically,

$$m(p, x) = C(u(x), p) \quad (5)$$

The money metric defines the minimum cost of buying bundles as least as good as  $x$ . The money metric utility function is sometimes called the "minimum income function" or the "direct compensation function".

**2.5. Money Metric Indirect Utility Function.** For the price vectors  $p$  and  $p^0$  and income  $m$ , the money metric indirect utility function is defined by

$$\mu(p, p^0, m) = c(\psi(p^0, m), p) \quad (6)$$

The indirect money metric utility function defines the minimum cost of buying bundles at prices  $p$  that yield utility at least as large as than obtained when prices are  $p^0$  and income is  $m$ . The money metric indirect utility function is sometimes called the indirect compensation function.

**2.6. Ordinary Demand.** An ordinary demand function specifies the optimal levels  $x$  for a given set of prices and income.

$$x(m, p) = \operatorname{argmax} [v(x) : px = m] \quad (7)$$

It is obtainable from the indirect utility function via Roy's identity.

$$x_i(m, p) = \frac{-\frac{\partial \psi [c(u, p), p]}{\partial p_i}}{\frac{\partial \psi [c(u, p), p]}{\partial m}} = \frac{-\frac{\partial \psi (u, p)}{\partial p_i}}{\frac{\partial \psi (u, p)}{\partial m}} \quad (8)$$

**2.7. Hicksian Demand.** An Hicksian or compensated demand function specifies the cost minimizing levels  $x$  for a given set of prices and specified utility level.

$$h(u, p) = \operatorname{argmin} [px : v(x) = u] \quad (9)$$

It is obtainable from the cost function via Shephard's lemma.

$$h_i(u, p) = \frac{\partial c(u, p)}{\partial p_i} \quad (10)$$

### 3. COMPENSATING AND EQUIVALENT VARIATION

**3.1. Ideal welfare change measures.** Consider two alternative states of the world. In the first state (or status quo), the consumer faces prices  $p^0$  and has income  $m^0$ . In the second state the consumer faces prices  $p^j$  and has income  $m^j$ . The utility maximizing consumer will obtain utility level  $\psi(m^0, p^0)$  in the initial state and utility level  $\psi(m^j, p^j)$  in state  $j$ . Here  $\psi(p, m)$  is the indirect utility function yielding the maximal level of utility with prices  $p$  and income  $m$ . If all we care about is which state is optimal, this measure is sufficient. Utility is an ordinal measure so the consumer is better or worse off as  $\psi(p^j, m^j)$  is greater than or less than  $\psi(p^0, m^0)$ . The difficulty with using this ordinal measure is that it allows no comparisons across individuals and gives no indication of the strength of preference for  $\{p^0, m^0\}$  versus  $\{p^j, m^j\}$ .

**3.2. Definitions of Compensating and Equivalent Variation.** Sir John R. Hicks [17, 18] proposed two “willingness to pay measures” to allow for monetary measurement of welfare change.

**Definition 1** (Equivalent Variation). Equivalent variation (EV) is defined as the amount of money paid to an individual with base prices and income that leads to the same satisfaction as that generated by a price and income change.

$$\begin{aligned} EV &= c(\psi(m^j, p^j), p^0) - c(\psi(m^0, p^0), p^0) \\ &= c(\psi(m^j, p^j), p^0) - m^0 \end{aligned} \quad (11)$$

If there is no change in income between the initial price and income pair and the subsequent one, we can also write EV as follows given that  $c(\psi(m, p^j), p^j) = c(\psi(m, p^0), p^0)$

$$\begin{aligned} EV &= c(\psi(m^j, p^j), p^0) - c(\psi(m^0, p^0), p^0) \\ &= c(\psi(m, p^j), p^0) - c(\psi(m, p^j), p^j) \end{aligned} \quad (12)$$

In equation 11,  $c(u, p)$  is the cost or expenditure function,  $\psi(m, p)$  is the indirect utility function,  $p^0$  is the initial price vector,  $p^j$  is any other price vector,  $m^0$  is initial income and  $m^j$  is subsequent income. Equivalent variation is the amount of money one has to give to a consumer so that she could attain the utility level possible with the new prices and income while facing base prices and only having base income. Equivalent variation can be thought of as the amount of money the consumer would accept in lieu of the price change. Equivalent variation measures the difference in attaining the initial utility level at the initial and subsequent prices. Equivalent variation is negative if the price and income change would make the consumer worse off.

**Definition 2** (Compensating Variation). Compensating variation (CV) measures the net revenue of a planner who must compensate the consumer for a price change after it occurs, bringing her back to the utility level  $\psi(m^0, p^0)$ . Compensating variation would be negative if the planner would have to pay the consumer a positive level of compensation because the price change makes her worse off. Compensating Variation is defined implicitly by

$$\begin{aligned} CV &= c(\psi(m^j, p^j), p^j) - c(\psi(m^0, p^0), p^j) \\ &= m^j - c(\psi(m^0, p^0), p^j) \end{aligned} \quad (13)$$

If there is no change in income between the initial price and income pair and the subsequent one, we can also write CV as follows given that  $c(\psi(m, p^j), p^j) = c(\psi(m, p^0), p^0)$

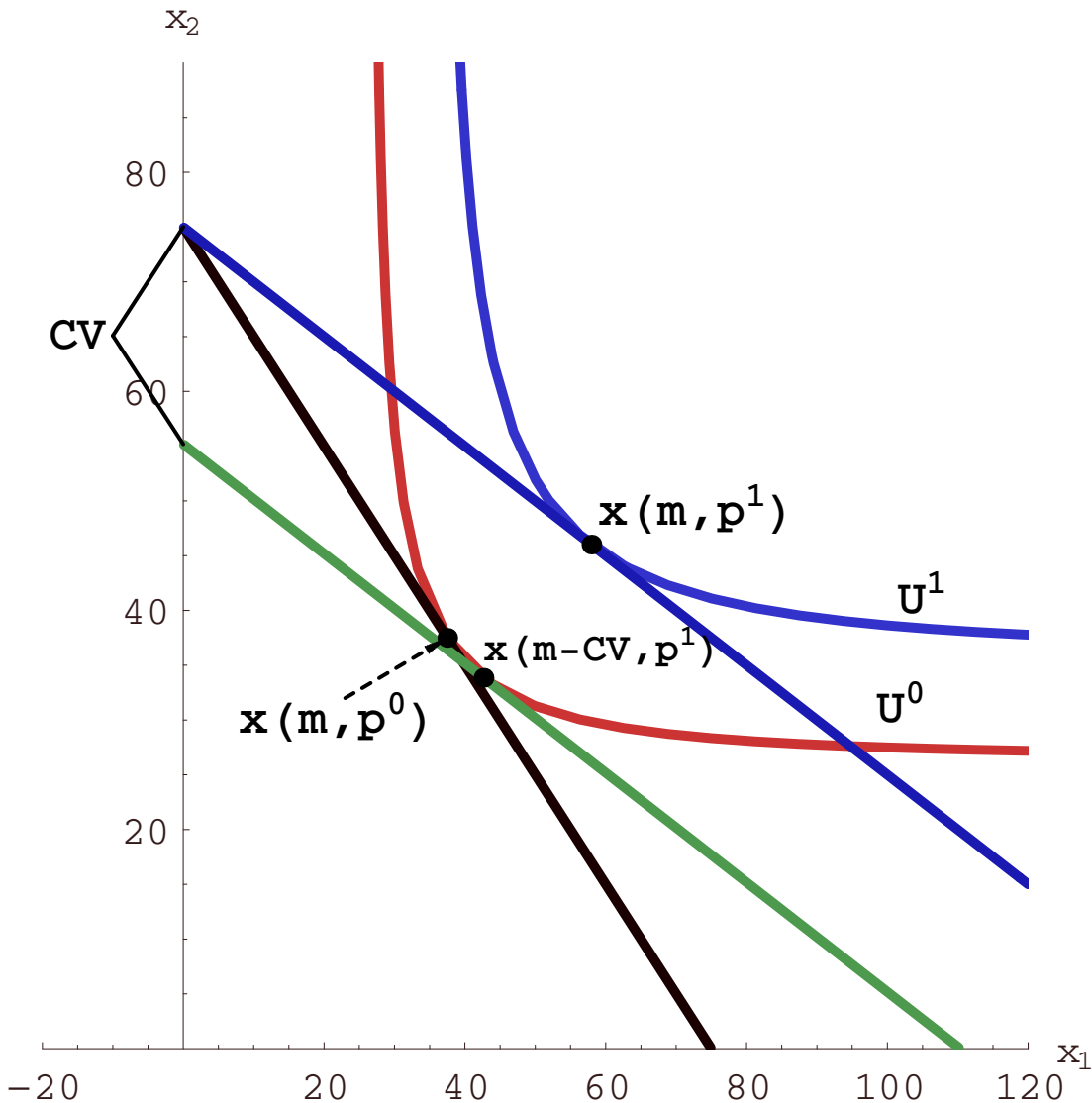
$$\begin{aligned} CV &= c(\psi(m, p^j), p^j) - c(\psi(m, p^0), p^j) \\ &= c(\psi(m, p^0), p^0) - c(\psi(m, p^0), p^j) \end{aligned} \quad (14)$$

Compensating variation is negative of the amount of money the consumer would be just willing to accept from the planner to allow the price change to take place. Compensating variation measures the difference in attaining the initial utility level at the initial and subsequent prices. Compensation takes place after the price and income change, so that compensating variation uses the after change prices.

**3.3. Graphical Analysis of Compensating Variation and Equivalent Variation.** Consider figure 1. At the prices and income  $(m, p^0)$  the optimal demand is at  $x(m, p^0)$  along the indifference curve  $U^0$ . As the price of good one falls, the budget line rotates outward and the consumer moves to point  $x(m, p^1)$  along the indifference curve  $U^1$ . Compensating variation measures the amount of income that must be taken away from the consumer at the new lower prices to leave her at the old utility level. At the new prices and this

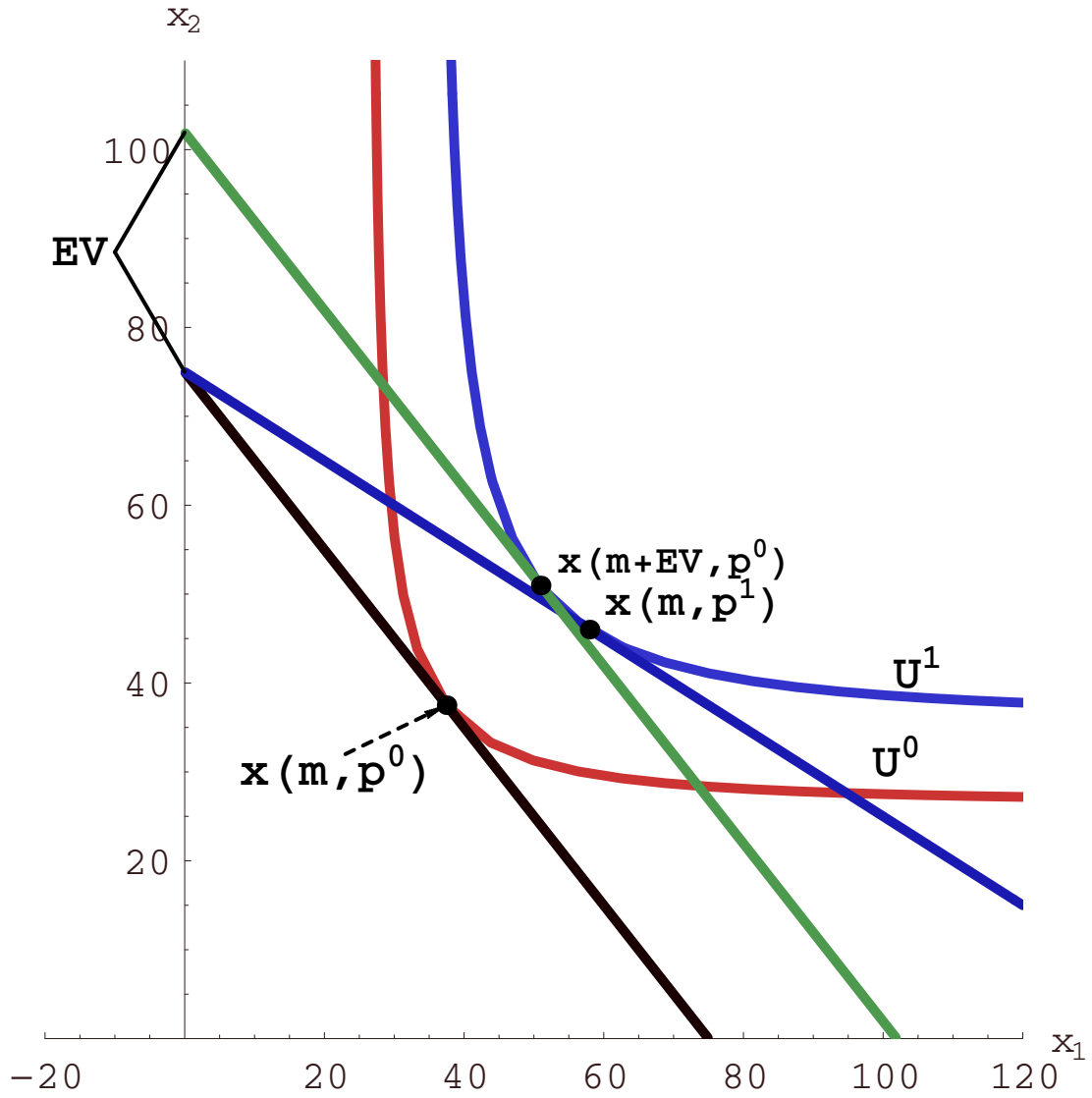
lower income level she consumes at point  $x(m-CV, p^1)$ . Compensating variation is the distance between the two budget lines along the vertical axis.

FIGURE 1. Compensating Variation



Consider figure 2. At the prices and income  $(m, p^0)$  the optimal demand is at  $x(m, p^0)$  along the indifference curve  $U^0$ . As the price of good one falls, the budget line rotates outward and the consumer moves to point  $x(m, p^1)$  along the indifference curve  $U^1$ . Equivalent variation measures the amount of income that must be given to the consumer at the old prices to make her as well off as with the lower prices. At the new prices and this higher income level she consumes at point  $x(m+EV, p^0)$ . Equivalent variation is the distance between the two budget lines along the vertical axis.

FIGURE 2. Equivalent Variation



3.4. **Compensating and Equivalent Variation and the Indirect Money Metric Utility Function.** For the price vectors  $p$  and a reference set  $\bar{p}$  and income  $m$ , the money metric indirect utility function is defined by

$$\mu(p, \bar{p}, m) = c(\psi(m, \bar{p}), p) \tag{15}$$

It represents the cost of attaining utility level  $\psi(m, \bar{p})$  when prices are  $p$ . Similarly,

$$\mu(p^1, \bar{p}, m) = c(\psi(m, \bar{p}), p^1) \tag{16a}$$

$$\mu(p^0, \bar{p}, m) = c(\psi(m, \bar{p}), p^0) \tag{16b}$$

If we choose  $p^0$  as the base set of prices, we obtain one indirect money metric utility function, if we choose  $p^1$  as the base set of prices, we obtain a different one indirect money metric utility function.

$$\mu(p^1, p^0, m) = c(\psi(m, p^0), p^1) \quad (17a)$$

$$\mu(p^0, p^0, m) = c(\psi(m, p^0), p^0) \quad (17b)$$

$$\mu(p^1, p^1, m) = c(\psi(m, p^1), p^1) \quad (17c)$$

$$\mu(p^0, p^1, m) = c(\psi(m, p^1), p^0) \quad (17d)$$

The definition of compensating variation then becomes

$$\begin{aligned} CV &= c(\psi(m, p^1), p^1) - c(\psi(m, p^0), p^1) \\ &= \mu(p^1, p^1, m) - \mu(p^1, p^0, m) \end{aligned} \quad (18)$$

while equivalent variation becomes

$$\begin{aligned} EV &= c(\psi(m, p^1), p^0) - c(\psi(m, p^0), p^0) \\ &= \mu(p^0, p^1, m) - \mu(p^0, p^0, m) \end{aligned} \quad (19)$$

**3.5. Compensating and Equivalent Variation and Hicksian Demand Curves.** We know from Shephard's lemma that

$$\frac{\partial c(u, p)}{\partial p_i} = h_i(u, p) \quad (20)$$

where  $h_i(u, p)$  is Hicksian demand function for the  $i$ th good. It depend on the utility level and the prices of all goods. Consider a situation with only the  $i$ th price changes and write  $h_i(u, p)$  as follows  $h_i(u, p_i, \tilde{p}_{-i})$  where  $\tilde{p}_{-i}$  represents all prices except  $p_i$ . Equivalent variation from a change in the  $i$ th price from  $p_i^0$  to  $p_i^1$  is defined as

$$\begin{aligned} EV &= c(\psi(m, p_i^1, \tilde{p}_{-i}), p_i^0, \tilde{p}_{-i}) - c(\psi(m, p_i^0, \tilde{p}_{-i}), p_i^0, \tilde{p}_{-i}) \\ &= c(\psi(m, p_i^1, \tilde{p}_{-i}), p_i^0, \tilde{p}_{-i}) - c(\psi(m, p_i^1, \tilde{p}_{-i}), p_i^1, \tilde{p}_{-i}) \end{aligned} \quad (21)$$

This can be written as an integral in the following manner,,

$$EV = \int_{p_i^1}^{p_i^0} h_i(\psi(m, p_i^1, \tilde{p}_{-i}), p_i, \tilde{p}_{-i}) dp_i \quad (22)$$

Thus the change in consumer welfare as measured by equivalent variation can be represented by the area lying between  $p_i^0$  and  $p_i^1$  and to the left of the Hicksian demand curve for good  $i$  associated with new utility level  $\psi(m, (p_i^1, \tilde{p}_{-i}^1))$ .

Similarly compensating variation from a change in the  $i$ th price from  $p_i^0$  to  $p_i^1$  is defined as

$$\begin{aligned} CV &= c(\psi(m, p_i^1, \tilde{p}_{-i}), p_i^1, \tilde{p}_{-i}) - c(\psi(m, p_i^0, \tilde{p}_{-i}), p_i^1, \tilde{p}_{-i}) \\ &= c(\psi(m, p_i^0, \tilde{p}_{-i}), p_i^1, \tilde{p}_{-i}) - c(\psi(m, p_i^0, \tilde{p}_{-i}), p_i^0, \tilde{p}_{-i}) \end{aligned} \quad (23)$$

This can be written as an integral in the following manner,,

$$CV = \int_{p_i^1}^{p_i^0} h_i(\psi(m, p_i^0, \tilde{p}_{-i}), p_i, \tilde{p}_{-i}) dp_i \quad (24)$$

Thus the change in consumer welfare as measured by compensating variation can be represented by the area lying between  $p_i^0$  and  $p_i^1$  and to the left of the Hicksian demand curve for good  $i$  associated with initial utility level  $\psi(m, (p_i^0, \tilde{p}_{-i}^0))$ .

For a single price change and a normal good, EV will be larger than CV for a fall in price and  $EV < CV$  for an increase in price. This relation changes then the good in question is an inferior good. If there are no wealth effects for a particular good, then EV and CV will be the same for a given price change. As will be discussed later, in this case, CV and EV and the consumer surplus will all be equivalent. For the example in figures 1 and 2, compensating variation is 79.3 while equivalent variation is 107.8.

Chipman and Moore [7] and McKenzie [24] have argued that compensating variation cannot be a valid measure of welfare change because it is not a monotonic, increasing transformation of the indirect utility function and thus preferences. They call for the use of equivalent variation. Their arguments are appropriate when considering all possible price changes and economic environments but many policy questions are specifically concerned with changes from the status quo. If consumers have a property right to the status quo then the appropriate welfare measure is compensating variation because the policymaker wants to know how much the consumer must be paid to give up that right.

**3.6. example problem.** Consider a consumer with the following utility function:  $v(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}}$ . To maximize utility subject to a budget constraint we obtain we set up a Lagrangian function.

$$\mathcal{L} = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} - \lambda [p_1 x_1 + p_2 x_2 - m] \quad (25)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{\frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{2}}}{x_1} - \lambda p_1 = 0 \quad (26a)$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{\frac{1}{2} x_1^{\frac{1}{4}} x_2^{-\frac{1}{2}}}{x_2} - \lambda p_2 = 0 \quad (26b)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -[p_1 x_1 + p_2 x_2] + m = 0 \quad (26c)$$

Take the ratio of the 26a and 26b to obtain

$$\begin{aligned} \frac{\frac{\frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{2}}}{x_1}}{\frac{\frac{1}{2} x_1^{\frac{1}{4}} x_2^{-\frac{1}{2}}}{x_2}} &= \frac{p_1}{p_2} \\ \Rightarrow \frac{x_2}{2 x_1} &= \frac{p_1}{p_2} \end{aligned} \quad (27)$$

We can now solve the equation for the quantity of good 2 as a function of the quantity of good 1 and the prices of both goods. Doing so we obtain

$$x_2 = \frac{2 x_1 p_1}{p_2} \quad (28)$$

Now substitute 28 in 26c to obtain

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \lambda} &= -[p_1 x_1 + p_2 x_2] + m = 0 \\
\Rightarrow p_1 x_1 + p_2 \frac{2 x_1 p_1}{p_2} &= m \\
\Rightarrow p_1 x_1 + 2 x_1 p_1 &= m \\
\Rightarrow 3 p_1 x_1 &= m \\
\Rightarrow x_1 &= \frac{m}{3 p_1}
\end{aligned} \tag{29}$$

Similarly for  $x_2$  so that we have

$$x_2 = \frac{2m}{3p_2} \tag{30}$$

We can find the indirect utility function  $\psi(m, p_1, p_2)$  by substitution

$$\begin{aligned}
\psi(m, p_1, p_2) &= x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} \\
&= \left( \frac{m}{3 p_1} \right)^{\frac{1}{4}} \left( \frac{2m}{3 p_2} \right)^{\frac{1}{2}} \\
&= \frac{\sqrt{2}}{3^{\frac{3}{4}}} \left( \frac{m}{p_1} \right)^{\frac{1}{4}} \left( \frac{m}{p_2} \right)^{\frac{1}{2}}
\end{aligned} \tag{31}$$

The cost function for this consumer is obtained by minimizing cost subject to a utility constraint. First set up the Lagrangian problem

$$\mathcal{L} = p_1 x_1 + p_2 x_2 - \lambda (x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} - u) \tag{32}$$

The first order conditions are as follows

$$\frac{\partial \mathcal{L}}{\partial x_1} = p_1 - \frac{1}{4} \frac{v}{x_1} \lambda = 0 \tag{33a}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = p_2 - \frac{1}{2} \frac{v}{x_2} \lambda = 0 \tag{33b}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} + u = 0 \tag{33c}$$

Taking the ratio of the equations 33a and 33b we obtain

$$\frac{p_1}{p_2} = \frac{x_2}{2x_1} \tag{34}$$

We can then solve for  $x_2$  as

$$x_2 = \frac{2 x_1 p_1}{p_2} \tag{35}$$

Substituting in the equation 33c we obtain



$$\begin{aligned}
u &= x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} \\
&= x_1^{\frac{1}{4}} \left( \frac{2 x_1 p_1}{p_2} \right)^{\frac{1}{2}} \\
&= 2^{\frac{1}{2}} \left( \frac{p_1}{p_2} \right)^{\frac{1}{2}} x_1^{\frac{3}{4}}
\end{aligned} \tag{36}$$

Solving for  $x_1$  we obtain

$$\begin{aligned}
u &= 2^{\frac{1}{2}} \left( \frac{p_1}{p_2} \right)^{\frac{1}{2}} x_1^{\frac{3}{4}} \\
\Rightarrow x_1^{\frac{3}{4}} &= u 2^{-\frac{1}{2}} \left( \frac{p_2}{p_1} \right)^{\frac{1}{2}} \\
\Rightarrow x_1 &= u^{\frac{4}{3}} 2^{-\frac{2}{3}} \left( \frac{p_2}{p_1} \right)^{\frac{2}{3}} \\
&= u^{\frac{4}{3}} \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( \frac{p_2}{p_1} \right)^{\frac{2}{3}}
\end{aligned} \tag{37}$$

Similarly for  $x_2$

$$x_2 = u^{\frac{4}{3}} 2^{\frac{1}{3}} \left( \frac{p_1}{p_2} \right)^{\frac{1}{3}} \tag{38}$$

Now if we substitute for the  $x_1$  and  $x_2$  in the cost expression we obtain

$$\begin{aligned}
c &= p_1 x_1 + p_2 x_2 \\
&= p_1 \left( u^{\frac{4}{3}} \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( \frac{p_2}{p_1} \right)^{\frac{2}{3}} \right) + p_2 \left( u^{\frac{4}{3}} 2^{\frac{1}{3}} \left( \frac{p_1}{p_2} \right)^{\frac{1}{3}} \right) \\
&= u^{\frac{4}{3}} \left( \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \right) + 2^{\frac{1}{3}} \left( p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \right) \right) \\
&= u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \left( \left( \frac{1}{2} \right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \right) \\
&= u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \left( 2^{-\frac{2}{3}} + 2^{\frac{1}{3}} \right) \\
&= u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} 2^{-\frac{2}{3}} \left( 2^{-\frac{2}{3}} 2^{\frac{2}{3}} + 2^{\frac{1}{3}} 2^{\frac{2}{3}} \right) \\
&= u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} 2^{-\frac{2}{3}} (1 + 2) \\
&= \frac{3}{2^{\frac{2}{3}}} u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}}
\end{aligned} \tag{39}$$

How consider two sets of prices with the same income. Specifically,

$$\{p^0, m^0\} = \{4, 4, 300\}$$

$$\{p^1, m^0\} = \{2, 4, 300\}$$

We compute indirect utility in the initial situation as follows

$$\begin{aligned}
 \psi_0(p^0, m^0) &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} \left(\frac{m}{p_1}\right)^{\frac{1}{4}} \left(\frac{m}{p_2}\right)^{\frac{1}{2}} \\
 &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} \left(\frac{300}{4}\right)^{\frac{1}{4}} \left(\frac{300}{4}\right)^{\frac{1}{2}} \\
 &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} 75^{\frac{3}{4}} \\
 &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} 3^{\frac{3}{4}} 25^{\frac{3}{4}} \\
 &= \sqrt{2} 25^{\frac{3}{4}} \\
 &= \sqrt{2} 25^{\frac{1}{2}} 25^{\frac{1}{4}} \\
 &= 5\sqrt{2}\sqrt{5} \\
 &= 5\sqrt{10}
 \end{aligned} \tag{40}$$

We compute indirect utility in the subsequent situation as follows

$$\begin{aligned}
 \psi_1(p^1, m^1) &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} \left(\frac{m}{p_1}\right)^{\frac{1}{4}} \left(\frac{m}{p_2}\right)^{\frac{1}{2}} \\
 &= \frac{\sqrt{2}}{3^{\frac{3}{4}}} \left(\frac{300}{2}\right)^{\frac{1}{4}} \left(\frac{300}{4}\right)^{\frac{1}{2}} \\
 &= \frac{2^{\frac{3}{4}}}{3^{\frac{3}{4}}} 2^{\frac{1}{3}} 75^{\frac{3}{4}} \\
 &= \frac{2^{\frac{3}{4}}}{3^{\frac{3}{4}}} 3^{\frac{3}{4}} 25^{\frac{3}{4}} \\
 &= 2^{\frac{3}{4}} 5\sqrt{5}
 \end{aligned} \tag{41}$$

Now compute the various measures of cost. Before doing so note that

$$\begin{aligned}
\psi_0(p^0, m^0)^{\frac{4}{3}} &= \left(5\sqrt{10}\right)^{\frac{4}{3}} \\
&= 5^{\frac{4}{3}} 5^{\frac{2}{3}} 2^{\frac{2}{3}} \\
&= 2^{\frac{2}{3}} 25 \\
\psi_1(p^1, m^1)^{\frac{4}{3}} &= \left(2^{\frac{3}{4}} 5\sqrt{5}\right)^{\frac{4}{3}} \\
&= 2 \left(5^{\frac{2}{3}} 5^{\frac{4}{3}}\right) = 50
\end{aligned} \tag{42}$$

Now for the cost of obtaining utility level  $\psi_1$  at prices  $p_1$ . First find the cost of obtaining utility level  $\psi_1$  at prices  $p^1$ .

$$\begin{aligned}
c(\psi_1, p^1) &= \frac{3}{2^{\frac{2}{3}}} u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \\
&= \frac{3}{2^{\frac{2}{3}}} 50 2^{\frac{1}{3}} 4^{\frac{2}{3}} \\
&= 150 \times 2^{-\frac{1}{3}} \times 4^{\frac{2}{3}} \\
&= 150 \times 2^{-\frac{1}{3}} \times 2^{\frac{4}{3}} \\
&= 150 \times 2 = 300
\end{aligned} \tag{43}$$

Now find the cost of obtaining utility level  $\psi_1$  at prices  $p^0$ .

$$\begin{aligned}
c(\psi_1, p^0) &= \frac{3}{2^{\frac{2}{3}}} u^{\frac{4}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \\
&= \frac{3}{2^{\frac{2}{3}}} 50 4^{\frac{1}{3}} 4^{\frac{2}{3}} \\
&= \frac{3 \times 50 \times 4}{2^{\frac{2}{3}}} \\
&= 600 \times 2^{-\frac{2}{3}} \\
&= 600 \times 2^{-1} \times 2^{\frac{1}{3}} \\
&= 300 \times 2^{\frac{1}{3}}
\end{aligned} \tag{44}$$

Similarly for the other costs.

$$\begin{aligned}
c(\psi_1, p^1) &= 300 \\
c(\psi_1, p^0) &= 300 \times 2^{\frac{1}{3}} \\
c(\psi_0, p^0) &= 300 \\
c(\psi_0, p^1) &= 150 \times 2^{\frac{2}{3}}
\end{aligned} \tag{45}$$

Equivalent variation is given by

$$\begin{aligned}
EV &= c(\psi(m^1, p^1), p^0) - c(\psi(m^0, p^0), p^0) \\
&= (300 \times 2^{\frac{1}{3}}) - 300 \\
&= 300 \left( 2^{\frac{1}{3}} - 1 \right) \\
&= 77.7963
\end{aligned} \tag{46}$$

Compensating variation is given by

$$\begin{aligned}
CV &= c(\psi(m^1, p^1), p^1) - c(\psi(m^0, p^0), p^1) \\
&= 300 - (150 \times 2^{\frac{2}{3}}) \\
&= 61.8898
\end{aligned} \tag{47}$$

Consider the Hicksian demand for  $x_1$  from equation 37

$$x_1 = u^{\frac{4}{3}} \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( \frac{p_2}{p_1} \right)^{\frac{2}{3}} \tag{48}$$

Substituting into equation 48 the subsequent utility level and price of  $p_2$  we obtain

$$\begin{aligned}
x_1 &= u^{\frac{4}{3}} \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( \frac{p_2}{p_1} \right)^{\frac{2}{3}} \\
&= \left( 2^{\frac{3}{4}} 5 \sqrt{5} \right)^{\frac{4}{3}} 2^{\frac{-2}{3}} \left( \frac{4}{p_1} \right)^{\frac{2}{3}} \\
&= 50 2^{\frac{-2}{3}} 2^{\frac{4}{3}} p_1^{\frac{-2}{3}} \\
&= 50 2^{\frac{2}{3}} p_1^{\frac{-2}{3}}
\end{aligned} \tag{49}$$

Now integrate this from 2 to 4 to obtain equivalent variation

$$\begin{aligned}
EV &= \int_2^4 50 2^{\frac{2}{3}} p_1^{\frac{-2}{3}} dp_1 \\
&= 50 2^{\frac{2}{3}} \int_2^4 p_1^{\frac{-2}{3}} dp_1 \\
&= 50 2^{\frac{2}{3}} 3 p_1^{\frac{1}{3}} \Big|_2^4 \\
&= \left( 150 2^{\frac{2}{3}} \right) \left( 4^{\frac{1}{3}} - 2^{\frac{1}{3}} \right) \\
&= 300 \left( 2^{\frac{-1}{3}} \right) \left( 2^{\frac{2}{3}} - 2^{\frac{1}{3}} \right) \\
&= 300 \left( 2^{\frac{1}{3}} - 1 \right) \\
&= 77.9763
\end{aligned} \tag{50}$$

Substituting into equation 48 the initial utility level and price of  $p_2$  we obtain

$$\begin{aligned}
x_1 &= u^{\frac{4}{3}} \left(\frac{1}{2}\right)^{\frac{2}{3}} \left(\frac{p_2}{p_1}\right)^{\frac{2}{3}} \\
&= \left(5\sqrt{10}\right)^{\frac{4}{3}} 2^{-\frac{2}{3}} \left(\frac{4}{p_1}\right)^{\frac{2}{3}} \\
&= 2^{\frac{2}{3}} 25 2^{-\frac{2}{3}} 2^{\frac{4}{3}} p_1^{-\frac{2}{3}} \\
&= 50 \times 2^{\frac{1}{3}} \times p_1^{-\frac{2}{3}}
\end{aligned} \tag{51}$$

Now integrate this from 2 to 4 to obtain equivalent variation

$$\begin{aligned}
CV &= \int_2^4 50 2^{\frac{1}{3}} p_1^{-\frac{2}{3}} dp_1 \\
&= 50 2^{\frac{1}{3}} \int_2^4 p_1^{-\frac{2}{3}} dp_1 \\
&= 50 2^{\frac{1}{3}} 3 p_1^{\frac{1}{3}} \Big|_2^4 \\
&= \left(150 2^{\frac{1}{3}}\right) \left(4^{\frac{1}{3}} - 2^{\frac{1}{3}}\right) \\
&= 300 \left(2^{-\frac{2}{3}}\right) \left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right) \\
&= 300 \left(1 - 2^{-\frac{1}{3}}\right) \\
&= 61.8898
\end{aligned} \tag{52}$$

#### 4. CONSUMER SURPLUS

**4.1. Definition of consumer surplus.** Consider the demand for a product  $x_i(p, m)$  and a change in price from  $p_i^0$  and  $p_i^1$ . The area to the left of the ordinary demand curve for good  $i$  is called the change in consumer surplus [9] associated with the change in price from  $p_i^0$  and  $p_i^1$ . Mathematically it is given by

$$\Delta CS = \int_{p_i^1}^{p_i^0} x_i(p_i, \tilde{p}_{-i}, m) dp_i \tag{53}$$

Consider figure 3. The area to the left of the demand curve between the two prices of good  $i$  is equal to consumer surplus.

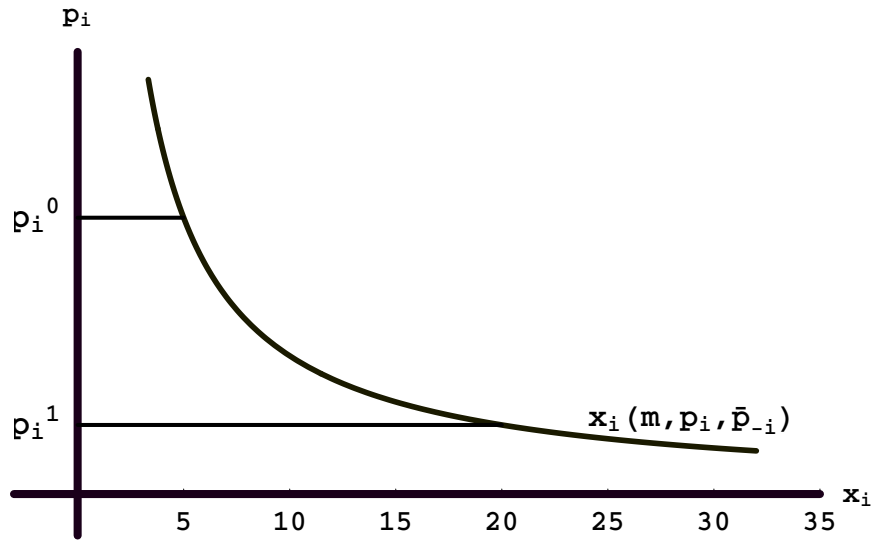
If preferences can be represented by a quasilinear indirect utility function

$$\psi(m, p) = g(p) + m \tag{54}$$

then compensating variation will be equal to equivalent variation and both are equal to the integral in equation 53. With other preferences, there will be a divergence between compensating variation, equivalent variation and consumer surplus. With quasilinear preferences, when there are no wealth effects for good  $i$ , the measure in equation 53 is referred to as Marshallian [22] consumer surplus.

When good  $i$  is a normal good, Marshallian consumer surplus overstates compensating variation and understates equivalent variation for both increases and decreases in  $p_i$ . If good  $i$  is an inferior good, Marshallian consumer surplus understates compensating variation and overstates equivalent variation for both

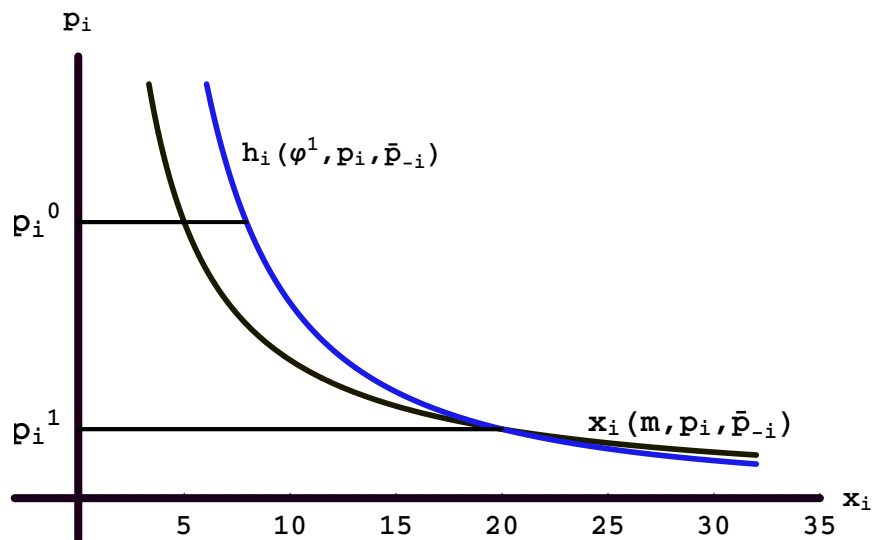
FIGURE 3. Consumer Surplus



increases and decreases in  $p_i$ . If the wealth effects for the goods in question are small, consumer surplus will be very close to both CV and EV. If the good being considered is one among many, because changes in real income will be spread around among many goods, the wealth effects will be small and the error from using consumer surplus instead of EV or CV will be small for the good in question.

Consider figure 4. Consumer surplus is less than EV which is measured at the higher utility level possible due to a decrease in  $p_i$ .

FIGURE 4. Consumer Surplus and Equivalent Variation



Consider figure 5. Consumer surplus is greater than CV which is measured at the initial utility level possible with the original price of  $p_i$ . Also consider figure 6 where consumer surplus is greater than CV and less than EV.

FIGURE 5. Consumer Surplus and Compensating Variation

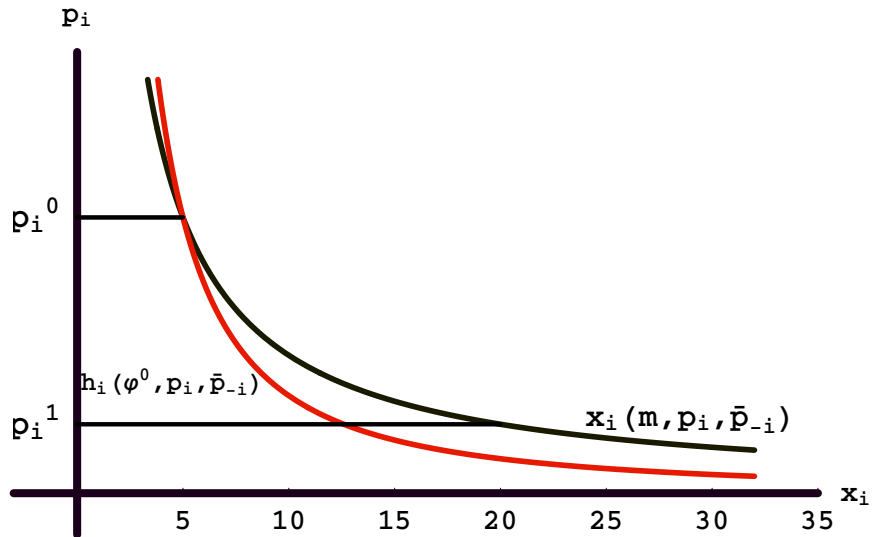
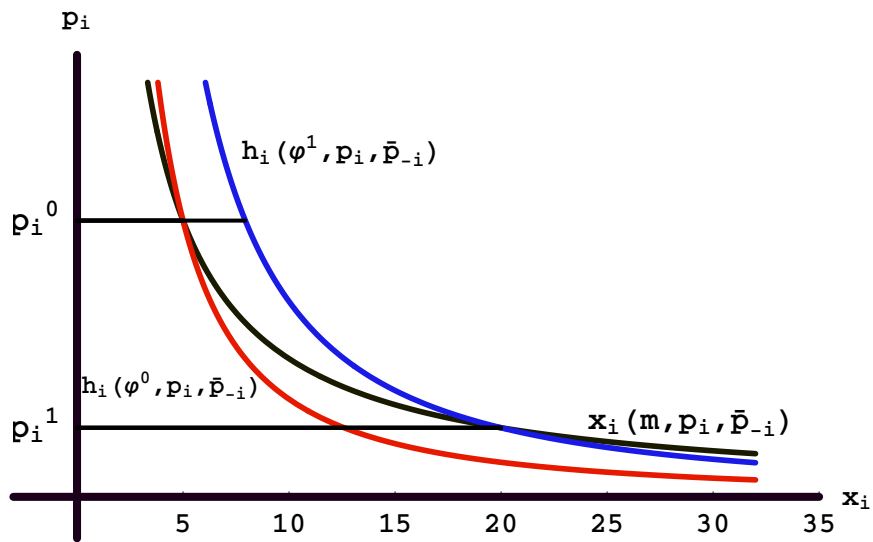


FIGURE 6. Consumer Surplus, Equivalent Variation and Compensating Variation



While the approximation error may be small for an individual commodity, when we are dealing with a large number of commodities, the aggregate error may become large. Hausman [15] has also pointed out that while the approximation error may be small as a percentage of the true welfare change, it may not be small as a percentage of *deadweight loss*.

## 4.2. Quasilinear Utility.

4.2.1. *Homothetic Preferences.* A preference relation is said to be homothetic if the slope of indifference curves remains constant along any ray through the origin. If preferences take this form, then knowing the shape of one indifference curve lets us know the shape of all indifference curves because they are "radial blowups" of each other. A preference relation is *homothetic* if for any two bundles  $x^1$  and  $x^2$  such that  $x^1 \sim x^2$ , then  $\alpha x^1 \sim \alpha x^2$  for any  $\alpha > 0$ . A preference relation  $\succeq$  is homothetic if and only if it can be represented by a utility function that is homogeneous of degree one.

4.2.2. *Quasilinear Preferences.* Consider a set of commodities  $x_0, x_1, x_2, \dots, x_n$ . Let the utility function have the following form.

$$v(x_0, x_1, x_2, \dots, x_n) = x_0 + \nu(x_1, x_2, \dots, x_n) \quad (55)$$

With these preferences, all indifference sets are parallel displacements of each other along the axis of commodity 0. Specifically, a preference relation is quasilinear if there is one commodity, called the *numeraire*, which shifts the indifference curves outward as consumption of it increases, without changing their slope. A preference relation is *quasilinear* if for any two bundles  $x^1$  and  $x^2$  such that  $x^1 \sim x^2$ , then  $(x^1 + \alpha e_0) \sim (x^2 + \alpha e_0)$  for  $e_0 = (1, 0, \dots, 0, \dots, 0)$  and any  $\alpha > 0$  where  $e_0$  is an  $n+1$  vector. We also assume that  $x + \alpha e_0 \succ x$  for all  $x$  and  $\alpha > 0$ . If we compute the marginal rate of substitution between any commodity and commodity 0, we obtain

$$MRS_{x_0 x_i} = -\frac{\frac{\partial v}{\partial x_i}}{\frac{\partial v}{\partial x_0}} = -\frac{\frac{\partial \nu}{\partial x_i}}{1} = \frac{\partial \nu}{\partial x_i} \quad (56)$$

Normalize the prices of all commodities such that  $p_0 = 1$  and consider the utility maximization problem.

$$\begin{aligned} \mathcal{L} &= v(x) - \lambda(x_0 + \sum_{i=1}^n p_i x_i - m) \\ &= x_0 + \nu(x_1, x_2, \dots, x_n) - \lambda(x_0 + \sum_{i=1}^n p_i x_i - m) \end{aligned} \quad (57)$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_0} = 1 - \lambda = 0 \quad (58a)$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial \nu}{\partial x_i} - \lambda p_i = 0, \quad i = 1, 2, \dots, n \quad (58b)$$

$$-x_0 - \sum_{i=1}^n p_i x_i + m = 0 \quad (58c)$$

From equation 58a,  $\lambda = 1$ . This then implies from equation 58b

$$\frac{\partial \nu}{\partial x_i} = p_i \quad (59)$$

or that the marginal utility of consumption of the  $i$ th good is equal to its price. This allows us to solve the  $n$  equations in 58b for the  $n$  demands as functions of the  $n$  prices. Thus the demand for the  $i^{\text{th}}$  commodity does not depend on income.

The indirect utility function for quasilinear preferences is of the form

$$\begin{aligned} \psi(m, p) &= m - \theta(p) \\ &= m + \phi(p) \end{aligned} \quad (60)$$



By Roy's identity, the demand for the  $i^{\text{th}}$  good is given by

$$\begin{aligned} x_i(m, p) &= \frac{-\frac{\partial \psi(m, p)}{\partial p_i}}{\frac{\partial \psi(m, p)}{\partial m}} \\ &= -\frac{\partial \phi(p)}{\partial p_i} \end{aligned} \quad (61)$$

**4.3. Approximating Compensating Variation for Small Price Changes.** Consider the situation when the change in the price of a good is reasonably small. The Hicksian demand function for good  $i$  at the intimal utility level is given by

$$\begin{aligned} h_i(\psi[m, (p_i^0, \tilde{p}_{-i})], p_i, \tilde{p}_{-i}) &= h_i(\psi^0, p_i, \tilde{p}_{-i}) \\ &= h_i(\psi^0, p) \end{aligned} \quad (62)$$

Compensating variation is obtained by integrating 62 from the initial level of  $p_i$  to the subsequent one. Consider a first order Taylor Series approximation to the Hicksian demand vector

$$\begin{aligned} \bar{h}(\psi[m, p^0], p) &= h(\psi[m, p^0], p^0) + \nabla_p h(\psi[m, p^0], p^0) (p - p^0) \\ &= h(\psi^0, p^0) + \nabla_p h(\psi^0, p^0) (p - p^0) \end{aligned} \quad (63)$$

Approximate compensating variation is obtained by integrating 63 from the initial level of  $p_i$  to the subsequent one or

$$\int_{p_i^1}^{p_i^0} \bar{h}_i(\psi^0, p_i, \tilde{p}_{-i}) dp_i \quad (64)$$

If the price change is small the linear approximation to Hicksian demand which has the same slope as the compensated demand at the initial price combination may provide a better approximation to CV than consumer surplus computed using ordinary demand. The relation between the slopes of the ordinary and Hicksian demand curves for normal goods is such that

$$\left| \frac{\partial h_i(u, p)}{\partial p_i} \right| < \left| \frac{\partial x_i(m, p)}{\partial p_i} \right| \quad (65)$$

Consider figure 7. In this case the linear approximation to Hicksian demand understates CV by less than consumer surplus overstates it.

We could also consider higher order approximations to Hicksian demand. The question is whether we have the information to compute  $\bar{h}(\psi[m, p^0], p)$ . The first term in the Taylor series expansion is just the level of demand at the initial prices and income. We know that at the initial point Hicksian and ordinary demand are the same so  $h(\psi^0, p^0) = x(m^0, p^0)$ . The Slutsky equation allows us to write Hicksian demand in terms of ordinary demand as follows

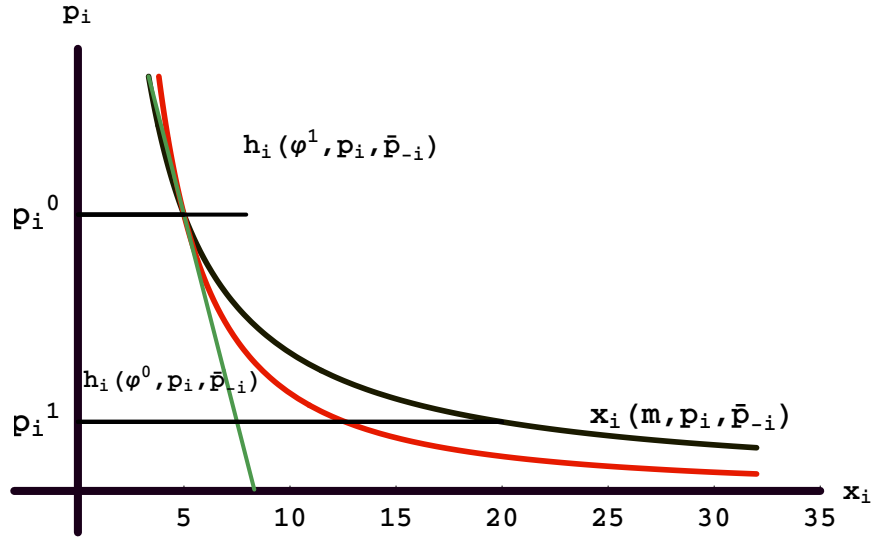
$$\frac{\partial h_i(u, p)}{\partial p_j} = \frac{\partial x_i(m, p)}{\partial p_j} + \frac{\partial x_i(m, p)}{\partial m} x_j(m, p) \quad (66)$$

For this specific case we obtain

$$\frac{\partial h_i(\psi[m, (p_i^0, \tilde{p}_{-i})], p_i, \tilde{p}_{-i})}{\partial p_i} = \frac{\partial x_i(m, p_i, \tilde{p}_{-i})}{\partial p_i} + \frac{\partial x_i(m, p_i, \tilde{p}_{-i})}{\partial m} x_i(m, p_i, \tilde{p}_{-i}) \quad (67)$$

Combining the information in equations 63, 64 and 67

FIGURE 7. Approximating Compensating Variation



$$\int_{p_i^1}^{p_i^0} \bar{h}_i(\psi^0, p_i, \tilde{p}_{-i}) dp_i = \int_{p_i^1}^{p_i^0} \left[ x_i(m^0, p_i^0, \tilde{p}_{-i}) + \left( \frac{\partial x_i(m^0, p_i^0, \tilde{p}_{-i})}{\partial p_i} + \frac{\partial x_i(m^0, p_i^0, \tilde{p}_{-i})}{\partial m} x_i(m^0, p_i^0, \tilde{p}_{-i}) \right) (p_i - p_i^0) \right] dp_i \quad (68)$$

All the terms in equation 68 are obtainable from information on ordinary demand. Note that  $x_i(m^0, p_i^0, \tilde{p}_{-i}^0)$ ,  $\frac{\partial x_i(m^0, p_i^0, \tilde{p}_{-i}^0)}{\partial p_i}$  and  $\frac{\partial x_i(m^0, p_i^0, \tilde{p}_{-i}^0)}{\partial m}$  are all evaluated at the initial prices and income and are more or less constants when we perform the integration. For small changes in price this will be a better approximation than consumer surplus, for larger changes, it is not clear which is better. But for larger changes we can always compute a higher order Taylor series approximation that will be closer than consumer surplus. A similar approach can be used to approximate equivalent variation.

McKenzie and Pearce, [23] in particular, have advocated the use of Taylor series approximations. The procedure is to estimate a system of demand equations that satisfy the standard integrability conditions (Hurwicz and Uzawa, [20]) and then use the derivatives of these estimated functions as parameters in a Taylor series approximation to equivalent variation or the money metric.

Applied researchers continue to use consumer surplus in hope that errors caused by non-constancy of the marginal utility of income are not large. Willig [36] formalized this approximation by developing bounds on consumer surplus that relate it directly to compensating and equivalent variation based on price and income elasticities. The bounds are obtained assuming that estimated ordinary demand functions are generated by utility maximizing consumers and using the money metric indirect utility function, that is  $c(\psi(m, \bar{p}), p)$  to develop an approximate relationship between consumer surplus and CV and EV. McKenzie and Pearce [23] have criticized the use such approximations and proposed direct calculation of the money metric or approximation via a Taylor series. Because a Taylor series can be made arbitrarily accurate, he argues that "Willig type" approximations are not necessary.

Vartia [34] has proposed an alternative method for calculating willingness to pay measures. He develops a way to computationally change prices while remaining on a given indifference surface. In effect he solves the differential equation

$$\frac{dc(t)}{dt} = \sum_{i=1}^n x_i(c(t), p(t)) \frac{dp_i(t)}{dt} \quad (69)$$

$$t \in [0,1], \quad c(0) = c^0$$

where  $x_i$  is the ordinary demand curve, and the vector of prices  $p$  varies over some path from the initial situation to a subsequent one. Specifically,  $p(t)$  is a differentiable curve in price space connecting  $p^0 = p(0)$  to  $p^1 = p(1)$  and  $c(t)$  is any expenditure path starting  $c^0 = c(0)$ . Using parameters of estimated ordinary demand functions Vartia's algorithm calculates  $c(\psi(m, \bar{p}), p)$  and  $h(\psi(m^0, \bar{p}), p)$  (the Hicksian demand curves) by solving the differential equation using numerical methods. The cost function or the Hicksian demand curves can be used to compute alternative welfare measures.

The most straightforward approach for obtaining these measures is to specify a functional form for the direct utility function and solve it for the implied demand equations to use in estimation. Changes in welfare can be evaluated using the implied indirect utility or expenditure function. Since the forms for which utility maximization leads to a closed form demand system are limited and restrictive, an alternative is to specify an indirect utility function and obtain demand equations using Roy's identity. The resulting system can be integrated to compute the appropriate welfare measures.

Each of the measurement methods discussed assumes estimated ordinary demand equations consistent with utility maximization by a consumer or profit maximization by a producer. The Willig approach is justified in few cases because if an estimated integrable system exists, exact as opposed to approximate methods are appropriate. If the system isn't integrable then the Willig approximations don't apply anyway.

The methods proposed by McKenzie and Vartia also require a system of integrable demand equations. The simplest way to obtain a system is to differentiate an indirect utility function, but this would lobby for the direct calculation of welfare change. These methods may allow for more general specifications of demand systems, where for example an indirect utility function may not have a closed form solution for the associated expenditure function. While this generalization is appealing in theory, determining and imposing these conditions in econometric estimation is often far from trivial. Furthermore, global imposition of these restrictions on arbitrary systems may add other undesired restrictions (Barnett [1]). While ability to impose and test restrictions is improving (Diewert and Wales [8]), estimation of systems derived from indirect utility functions seems the best current alternative.

## 5. AGGREGATION

The final issue is aggregation. Welfare measurement is of value to policymakers as it allows the aggregation of individual welfare. Even ignoring the issue of welfare weights, serious problems occur. While aggregation for producers under certainty is straightforward, the conditions for an aggregate demand system depending on prices and aggregate income to be consistent with individual demand systems generated by utility maximization are fairly restrictive (Gorman [10]). In particular, we know that for linear aggregation the indirect utility function for the  $i^{\text{th}}$  consumer is given by

$$\begin{aligned}\psi_i(p, m_i) &= \frac{m_i - f_i(p)}{g(p)} \\ &= \frac{1}{g(p)} m_i - \frac{f_i(p)}{g(p)}\end{aligned}\tag{70}$$

The aggregate indirect utility function is given by

$$\begin{aligned}\psi(p, m) &= \frac{m - \sum_{i=1}^I f_i(p)}{g(p)} \\ &= \frac{1}{g(p)} m - \frac{\sum_{i=1}^I f_i(p)}{g(p)}\end{aligned}\tag{71}$$

The term  $\sum_{i=1}^I f_i(p)$  just depends on prices and so could be written as  $\theta(p)$  which would give

$$\begin{aligned}\psi(p, m) &= \frac{1}{g(p)} m - \frac{\theta(p)}{g(p)} \\ &= \frac{1}{g(p)} m - \mu(p), \quad \mu(p) = \frac{\theta(p)}{g(p)}\end{aligned}\tag{72}$$

If preferences are quasilinear, then  $g(p) \equiv 1$  and we obtain  $\psi(p, m) = m - \mu(p)$  where  $\mu(p) = \theta(p)$  so that

$$\begin{aligned}\psi(p, m) &= m - \sum_{i=1}^I f_i(p) \\ &= m - \theta(p)\end{aligned}\tag{73}$$

Consider using Roy's identity to obtain demand from this indirect utility function.

$$\begin{aligned}\frac{\partial \psi(p, m)}{\partial p^k} &= - \sum_{i=1}^I \frac{\partial f_i(p)}{\partial p^k} \\ \frac{\partial \psi(p, m)}{\partial m} &= 1 \\ \Rightarrow x^k(m, p) &= \sum_{i=1}^I \frac{\partial f_i(p)}{\partial p^k} \\ &= \frac{\partial \theta(p)}{\partial p^k}\end{aligned}\tag{74}$$

This implies that we can integrate ordinary demand functions to get back the aggregate indirect utility function that represents the welfare of the aggregate of consumers.

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