

## MATHEMATICA EXERCISES

For all problems, write Mathematica code to find the solutions.

**Exercise 1.** Consider the following production function.

$$\begin{aligned} y &= f(x_1, x_2) \\ &= 30x_1 + 16x_2 - x_1^2 + x_1x_2 - 2x_2^2 \end{aligned}$$

- a. Write Mathematica code to define the function. This might look like.

$$f[x1_, x2_] := 30 * x1 + 16 * x2 - x1^2 + x1 * x2 - 2 * x2^2$$

- b. Find the marginal products of  $x_1$  and  $x_2$ .  
 c. Find the rate of technical substitution between  $x_1$  and  $x_2$ .  
 d. Find the Hessian matrix for this function.  
 e. Find the determinant of this Hessian matrix.  
 f. Find the elasticity of substitution for this function.

$$\sigma_{12} = \frac{-f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 (f_{11} f_2^2 - 2f_{12} f_1 f_2 + f_{22} f_1^2)}$$

- g. Find the elasticity of scale for this function.

$$\epsilon = \sum_{i=1}^n \frac{\partial f}{\partial x_i} \frac{x_i}{y}$$

- h. Write an equation for profit for a firm using this technology.

$$Profit[x1_, x2_] := p * f[x1, x2] - w1 * x1 - w2 * x2$$

- i. Find the derivatives of profit with respect to  $x_1$  and  $x_2$ .  
 j. Set these derivatives equal to zero and solve for the optimal levels of  $x_1$  and  $x_2$ .  
 k. What is a formula for the optimal level of output?  
 l. What are the own and cross price derivatives on input demand with respect to input price?  
 m. Now set  $p = 1$ ,  $w_1 = 10$ ,  $w_2 = 5$ . What are the optimal levels of input use. Do not solve the problem again.  
 n. What is optimal output?  
 o. What is profit?  
 p. Check the necessary and sufficient conditions for a maximum.  
 q. What is the rate of technical substitution at the optimal input levels? How does this relate to the input price ratio?  
 r. What is the elasticity of substitution at the optimal input levels?  
 s. What is the elasticity of scale at the optimal input levels?

**Exercise 2.** Consider the following production function.

$$\begin{aligned} y &= Ax_1^{\alpha_1} x_2^{\alpha_2} \\ &= 100 x_1^{\frac{2}{5}} x_2^{\frac{1}{5}} \end{aligned}$$

- Write Mathematica code to define the function.
- Find the marginal products of  $x_1$  and  $x_2$ .
- Find the rate of technical substitution between  $x_1$  and  $x_2$ .
- Find the Hessian matrix for this function.
- Find the determinant of this Hessian matrix.
- Find the elasticity of substitution for this function.

$$\sigma_{12} = \frac{-f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 (f_{11} f_2^2 - 2f_{12} f_1 f_2 + f_{22} f_1^2)}$$

- Find the elasticity of scale for this function.

$$\epsilon = \sum_{i=1}^n \frac{\partial f}{\partial x_i} \frac{x_i}{y}$$

- Write an equation for profit for a firm using this technology.

$$Profit[x1_, x2_] := p * f[x1, x2] - w1 * x1 - w2 * x2$$

- Find the derivatives of profit with respect to  $x_1$  and  $x_2$ .
- Set these derivatives equal to zero and solve for the optimal levels of  $x_1$  and  $x_2$ .
- What is a formula for the optimal level of output?
- What are the own and cross price derivatives on input demand with respect to input price?
- Now set  $p = 1$ ,  $w_1 = 10$ ,  $w_2 = 5$ . What are the optimal levels of input use. Do not solve the problem again.
- What is optimal output?
- What is profit?
- Check the necessary and sufficient conditions for a maximum.
- What is the rate of technical substitution at the optimal input levels? How does this relate to the input price ratio?
- What is the elasticity of substitution at the optimal input levels?
- What is the elasticity of scale at the optimal input levels?

**Exercise 3.** Consider the following production function.

$$\begin{aligned} y &= A \left[ \delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} \right]^{\frac{-\nu}{\rho}} \\ &= 100 \left[ 3/5 x_1^{-2} + 1/5 x_2^{-2} \right]^{\frac{-1/2}{2}} \end{aligned}$$

- Write Mathematica code to define the function.
- Find the marginal products of  $x_1$  and  $x_2$ .
- Find the rate of technical substitution between  $x_1$  and  $x_2$ .
- Find the Hessian matrix for this function.

- e. Find the determinant of this Hessian matrix.
- f. Find the elasticity of substitution for this function.

$$\sigma_{12} = \frac{-f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 (f_{11} f_2^2 - 2f_{12} f_1 f_2 + f_{22} f_1^2)}$$

- g. Find the elasticity of scale for this function.

$$\epsilon = \sum_{i=1}^n \frac{\partial f}{\partial x_i} \frac{x_i}{y}$$

- h. Now set  $p = 1$ ,  $w_1 = 10$ ,  $w_2 = 5$ . Write an equation for profit for a firm using this technology.

$$Profit[x1_, x2_] := p * f[x1, x1] - w1 * x1 - w2 * x2$$

- i. Find the derivatives of profit with respect to  $x_1$  and  $x_2$ .
- j. Set these derivatives equal to zero and solve for the optimal levels of  $x_1$  and  $x_2$ . You will probably need to use NSolve. What is optimal output?
- o. What is profit?
- p. Check the necessary and sufficient conditions for a maximum.
- q. What is the rate of technical substitution at the optimal input levels? How does this relate to the input price ratio?
- r. What is the elasticity of substitution at the optimal input levels?
- s. What is the elasticity of scale at the optimal input levels?

**Exercise 4.** Consider the following production function.

$$\begin{aligned} y &= A, x_1^{\alpha_1} x_2^{\alpha_2} e^{\beta_{11} \ln x_1 + \beta_{12} \ln x_1 \ln x_2 + \beta_{22} \ln x_2^2} \\ &= 5 x_1^{1/3} x_2^{1/2} e^{-.02 \ln x_1 + 0.1 \ln x_1 \ln x_2 + -0.2 \ln x_2^2} \end{aligned}$$

- a. Write Mathematica code to define the function.
- b. Find the marginal products of  $x_1$  and  $x_2$ .
- c. Find the rate of technical substitution between  $x_1$  and  $x_2$ .
- d. Find the Hessian matrix for this function.
- e. Find the determinant of this Hessian matrix.
- f. Find the elasticity of substitution for this function.

$$\sigma_{12} = \frac{-f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 (f_{11} f_2^2 - 2f_{12} f_1 f_2 + f_{22} f_1^2)}$$

- g. Find the elasticity of scale for this function.

$$\epsilon = \sum_{i=1}^n \frac{\partial f}{\partial x_i} \frac{x_i}{y}$$

- h. Now set  $p = 10$ ,  $w_1 = 2$ ,  $w_2 = 1$ . Write an equation for profit for a firm using this technology.

$$Profit[x1_, x2_] := p * f[x1, x1] - w1 * x1 - w2 * x2$$

- i. Find the derivatives of profit with respect to  $x_1$  and  $x_2$ .
- j. Use FindMaximum to find the optimal levels of input.

- k. What is optimal output?
- o. What is profit?
- p. Check the necessary and sufficient conditions for a maximum.
- q. What is the rate of technical substitution at the optimal input levels? How does this relate to the input price ratio?
- r. What is the elasticity of substitution at the optimal input levels?
- s. What is the elasticity of scale at the optimal input levels?