EXERCISE 2: INDIVIDUAL (14 Points Total)  L. Tesfatsion
DUE: Friday, September 8, 12:10pm.  Econ 502/Fall 2017

**CAUTION: Late assignments will not be accepted – no exceptions.**

**Note 1:** Please make an EXTRA copy of your exercise to bring to class on the due date for use in class discussion after you turn in your exercise.

**Note 2:** Students are permitted to work together in study groups on this exercise, but each student is asked to separately prepare and turn in their exercise answer. The techniques covered in this exercise are essential tools for subsequent exercises and exams, so “free riding” on other people’s work should definitely be avoided!

Effects of an Increase in the Income Tax Rate

Reference Materials:

- [3] The Basic Short-Run IS-LM Model with Sticky Prices, Syllabus II.A (COURSE PACKET 6)

Consider an economy represented in terms of an IS-LM model, hereafter referred to as model MOD, that consists of the following six model equations plus classification of variables:

Model Equations:

(E1) \( Y = C + I + G + NE \)

(E2) \( C = a + b[1 - t]Y \)

(E3) \( I = e - dR \)

(E4) \( NE = g - mY - nR \)

(E5) \( G - tY = D \)

(E6) \( M/P = kY - h[R + INF] \)
Classification of Variables:

Endogenous Variables:

\[ Y = \text{real income} \]

\[ C = \text{real consumption} \]

\[ I = \text{real gross investment} \]

\[ R = \text{real interest rate on bonds, i.e., nominal bond interest rate minus the inflation rate} \]

\[ NE = \text{real net exports} \]

\[ G = \text{real government expenditure} \]

Admissible Exogenous Variables:

\[ D = \text{positive target level for the real government budget deficit} \]

\[ t = \text{income tax rate with } 0 < t < 1 \]

\[ P = \text{positive price level} \]

\[ M = \text{positive nominal money supply} \]

\[ INF = \text{positive inflation rate} \]

\[ b = \text{marginal propensity to consume satisfying } 0 < b < 1 \]

\[ m = \text{marginal propensity to import satisfying } 0 < m < 1 \]

\[ a, e, d, g, n, k, h = \text{positive constants} \]

DEFINITION: For any specification of admissible values for model MOD’s exogenous variables, a solution for model MOD consists of values \((Y^o, C^o, I^o, R^o, NE^o, G^o)\) for model MOD’s six endogenous variables that satisfy the six model equations (E1)-(E6).

IMPORTANT CAUTION: Model MOD is not identical to the IS-LM model analyzed in Packet 6, hence the IS and LM equations you derive below for model MOD could differ from the IS and LM equations derived in Packet 6.
QUESTION 1 [4 Points Total]

Part Q1.A: (1 Point) Provide a carefully expressed economic definition of an IS equation for an aggregated macroeconomic model such as model MOD. That is, carefully explain the economic meaning of an IS equation for such a model (not simply its mathematical form).

Part Q1.B (1 Points): Proceeding carefully, using the general steps outlined in Course Packet 6, derive the IS equation for model MOD. Show your work.

Part Q1.C (1 Point): Construct a carefully labeled and carefully justified graph that depicts the form of the model MOD IS equation in the $Y$-$R$ plane (i.e., that depicts the “IS Curve” for model MOD).

Part Q1.D (1 Point): Using your answer for Q1.C, determine graphically how the position of the model MOD IS Curve in the $Y$-$R$ plane is affected by an increase in the income tax rate $t$. Justify your findings with care.

QUESTION 2 [4 Points Total]

Part Q2.A: (1 Point) Provide a carefully expressed economic definition of an LM equation for an aggregated macroeconomic model such as model MOD. That is, carefully explain the economic meaning of an LM equation for such a model (not simply its mathematical form).

Part Q2.B: (1 Point) Proceeding carefully, using the general steps outlined in Course Packet 6, derive the LM equation for model MOD. Show your work.

Part Q2.C: (1 Point) Construct a carefully labeled and carefully justified graph that depicts the form of the model MOD LM equation in the $Y$-$R$ plane (i.e., that depicts the “LM Curve” for model MOD).

Part Q2.D: (1 Point) Using your answer for Q2.C, determine graphically how the position of the model MOD LM Curve in the $Y$-$R$ plane is affected by an increase in the income tax rate $t$. Justify your findings with care.
QUESTION 3 [6 Points Total]

Part Q3.A [2 Point] Define a (reduced-form) solution for model MOD to be a point \((Y^o, R^o)\) in the \(Y-R\) plane that satisfies both the IS and LM equations for model MOD. Using your findings for questions Q1 and Q2, establish graphically that model MOD has a unique (reduced-form) solution \((Y^o, R^o)\).

Part Q3.B [2 Points] Using your findings for questions Q1 and Q2, determine graphically (if possible) how the model MOD (reduced-form) solution values \((Y^o, R^o)\) depicted in Part Q3.A are affected by an increase in the income tax rate \(t\). That is, for each of these solution values, determine whether it increases, stays the same, or decreases in value as \(t\) increases, or whether the direction of change in this value as a result of an increase in \(t\) cannot be determined without additional information. Justify your assertions carefully.

Part Q3.C [2 Points] Using your findings for question Q3.B, determine (if possible) how the model MOD solution values \(I^o\) and \(NE^o\) for \(I\) and \(NE\) are affected by an increase in the income tax rate \(t\). That is, for each of these solution values, determine whether it increases, stays the same, or decreases in value as \(t\) increases, or whether the direction of change in this value as a result of an increase in \(t\) cannot be determined without additional information. Justify your assertions carefully.