Effects of a Decrease in the Government Budget Deficit
in a Dynamic Sticky-Price IS-LM Model

Background Materials:

- Packet Materials on the Dynamic Sticky-Price IS-LM Model (PACKETS 6-8);
- “Dynamic Sticky-Price IS-LM Model: Illustrative Exercise” (PACKET 9)
- Answer Key for Exercise 3 (handed out in class)

Packet 9 provides an illustrative exercise for a modified version (“Model M”) of the dynamic sticky-price IS-LM model developed in Packets 6-8. A four-diagram graphical analysis is used to determine (as far as possible) the effects of an increase in the government budget deficit \( D \) at the beginning of period 1 on the Model M solution values for endogenous variables in time periods 1 and 2, assuming that the Model M economy begins in period 1 (prior to the increase in \( D \)) with actual real GDP less than potential real GDP, and that the increase in \( D \) at the beginning of period 1 is then maintained throughout all periods \( T \geq 1 \).

This exercise asks you to use a similar four-diagram graphical analysis to analyze the effects of a decrease in the government budget deficit \( D \) at the beginning of period 1 on Model M solution values for endogenous variables in time periods 1 and 2, assuming that the Model M economy begins in period 1 (prior to the decrease in \( D \)) with actual real GDP less than potential real GDP, and that the decrease in \( D \) at the beginning of period 1 is then maintained throughout all periods \( T \geq 1 \).
Model M: For each time period $T \geq 1$:

\begin{align*}
(1) & \quad Y(T) = C(T) + I(T) + G(T) + NE(T) \\
(2) & \quad C(T) = a + b[1 - t]Y(T) \\
(3) & \quad I(T) = e - dR(T) \\
(4) & \quad NE(T) = g - mY(T) - nR(T) \\
(5) & \quad G(T) - tY(T) = D \\
(6) & \quad M/P(T) = kY(T) - hR(T) \\
(7) & \quad N^*(T) = (1 + u)^T \cdot H([1 - t]w^*(T)) \\
(8) & \quad Y^*(T) = A(T)F(N^*(T), K(T)) \\
(9) & \quad w^*(T) = A(T)F_N(N^*(T), K(T)) \\
(10) & \quad A(T) = [1 + \theta \cdot G(T)] \\
(11) & \quad \pi(T, T + 1) = [P(T + 1) - P(T)]/P(T) \\
(12) & \quad \pi(T, T + 1) = \lambda \cdot \pi(T - 1, T) + f \cdot [Y(T) - Y^*(T)]/Y^*(T) \\
(13) & \quad K(T + 1) = I(T) + K(T)
\end{align*}

Classification of Variables and Admissibility Conditions:

Thirteen Period-T Endogenous Variables ($T \geq 1$):
$Y(T), C(T), I(T), G(T), NE(T), R(T), N^*(T), w^*(T), Y^*(T), A(T), \pi(T, T + 1), P(T + 1), K(T + 1)$

Three Period-T Predetermined (State) Variables ($T > 1$):
$\pi(T - 1, T), P(T), K(T)$

Admissible Exogenous Variables and Functional Forms:

Initial Conditions for State Variables ($T = 1$): $\pi^\alpha(0, 1) = 0, P^\alpha(1) > 0, K^\alpha(1) > 0$

Coefficients: $0 < a, 0 < e, 0 < d, 0 < g, 0 < n, 0 < k, 0 < h, 0 < u, 0 < \theta, 0 < \lambda, 0 < f, 0 < b < 1, 0 < m < 1$

Government Policy Variables: $0 < t < 1; 0 < D^\alpha; 0 < M$

Functional Forms:

$H(z)$ with $H(0) = 0$ and $dH(z)/dz > 0$ for all $z = [1 - t]w \geq 0$

$F(N, K)$ with $F(0, K) = F(N, 0) = 0, F_N > 0, F_K > 0, F_{NN} < 0, F_{KK} < 0, F_{NK} > 0$
Suppose, first, that the government maintains the government budget deficit at level $D^o$ for all periods $T \geq 1$. For any $T \geq 1$, let the resulting solution values for the 13 period-$T$ endogenous variables for the Model M equations (1)-(13) with deficit $D^o$ be referred to as the \textit{benchmark solution for period} $T$, denoted by

$$\text{BenchSol}_T = Y^o(T), C^o(T), I^o(T), G^o(T), NE^o(T), R^o(T), N^{*o}(T), w^{*o}(T), Y^{*o}(T), A^o(T), \pi^o(T), P^o(T + 1), K^o(T + 1)$$

Suppose, instead, that the government at the beginning of period 1 \textit{decreases} the government budget deficit from $D^o$ to a lower (but still positive) value $D'$ and then maintains the deficit at this lower value $D'$ for all periods $T \geq 1$. For any period $T \geq 1$, let the resulting solution values for the 13 period-$T$ endogenous variables for the Model M equations (1)-(13) with deficit $D'$ be referred to as the \textit{new solution for period} $T$, denoted by

$$\text{NewSol}_T = Y'(T), C'(T), I'(T), G'(T), NE'(T), R'(T), N^{*'}(T), w^{*'}(T), Y^{*'}(T), A'(T), \pi'(T), P'(T + 1), K'(T + 1)$$

***IMPORTANT NOTES ON EXERCISE 4***:

- Assume in your answers for all parts of Exercise 4 that, for the benchmark solution BenchSol1 for period 1, actual real GDP in period 1 is strictly lower than potential real GDP, and investment is strictly positive; that is, assume $Y^o(1) < Y^{*o}(1)$ and $0 < I^o(1)$.

- Observe that the first five equations for Model M are the same as the first five equations for the IS-LM model from Exercise 3. You should make use of this fact in Exercise 4 to save yourself some calculations.

- As in Packet 9, to simplify the analysis it is assumed in equation (6) for Model M that there is no “Keynes ex ante effect,” that is, it is assumed that real money demand depends on the \textit{real} interest rate $R$ rather than on the (\textit{ex ante}) nominal interest rate $R + \pi$.

- As in Packet 9, it is assumed in equation (10) for Model M that the Total Factor Productivity (TFP) coefficient $A(T)$ appearing in the period-$T$ aggregate production function is an increasing function of period-$T$ government expenditure $G(T)$, where $G(T)$ is classified as a period-$T$ endogenous variable. Thus, in Model M, both $A(T)$ and $G(T)$ are period-$T$ endogenous variables.
QUESTION 1 [13 POINTS TOTAL]:

Part Q1.A [6.5 Points]: Using a careful 4-diagram graphical analysis as illustrated in Packet 9, determine for each of the 13 period-1 endogenous variables whether its NewSol1 solution value resulting under the smaller deficit level $D'$ is larger, the same, smaller, or indeterminate in size in relation to its corresponding BenchSol1 solution value under the original deficit level $D^o$.

For example, for the particular endogenous variable $Y$ (real GDP), can you sign the difference $[Y'(1) - Y^o(1)]$? Why or why not? Be sure all of your graphs are carefully labeled and explained, and that all of your assertions are carefully justified in terms of Model M assumptions.

Part Q1.B [6.5 Points]: Provide a careful economic interpretation for your findings in Part Q1.A. That is, carefully discuss what you believe to be the economic meaning of these findings.

QUESTION 2 [13 POINTS TOTAL]:

Part Q2.A [6.5 Points]: Using a careful 4-diagram graphical analysis as illustrated in Packet 9, determine for each of the 13 period-2 endogenous variables whether its NewSol2 solution value resulting under the smaller deficit level $D'$ is larger, the same, smaller, or indeterminate in size in relation to its corresponding BenchSol2 solution value under the original deficit level $D^o$.

For example, for the particular endogenous variable $Y$ (real GDP), can you sign the difference $[Y'(2) - Y^o(2)]$? Why or why not? Be sure all of your graphs are carefully labeled and explained, and that all of your assertions are carefully justified in terms of Model M assumptions.

Part Q2.B [6.5 Points]: Provide a careful economic interpretation for your findings in Part Q2.A. That is, carefully discuss what you believe to be the economic meaning of these findings.

QUESTION 3 [4 POINTS]:

Based on your findings for Q1 and Q2, to what extent (if any) does Model M provide support for the following assertion currently being made by U.S. and European austerity advocates:

All else equal, a decrease in the government budget deficit (with no change in income tax rates) will result in a higher level of real GDP over the longer run.