EXERCISE 4: INDIVIDUAL (20 Points Total)  
L. Tesfatsion  
DUE: 12:10pm, Friday, September 22, 2017 Econ 502/Fall 2017

**CAUTION: Late assignments will not be accepted – no exceptions.**

Effects of Income Tax Rate Changes in a Solow-Swan Descriptive Growth Model

Background Materials:


**[2] “Notes on Differential Equations”, Sections on existence, uniqueness, and stability of stationary solutions; Syllabus III.B (PACKET 13);**

**[3] “The Basic Solow-Swan Descriptive Growth Model” Syllabus III.C (ONLINE)**

**VERY IMPORTANT NOTE:** All five parts of Exercise 4 should be answered using the particular version of the Solow-Swan Descriptive Growth Model appended at the end of this exercise in both level form (model MM) and per-capita form (model PCMM). The Exercise 4 models MM and PCMM differ in several key respects from the standard Solow-Swan descriptive growth model presented and analyzed in [3] in both level and per-capita form.

**PART A [5 Points]:** Explain briefly but carefully the economic meaning of each of the four model MM equations. Also, discuss the economic meaning of model MM as a whole (equations plus classification of variables); that is, what type of macroeconomy does model MM describe?

**PART B [4 Points]:** Using ref. [3] as a guide, establish carefully that model MM, which has two state variables K and L, can be reduced down to model PCMM, a model which has only one state variable k = K/L. Justify all your steps carefully. In particular, be sure to explain carefully in your answer how the admissibility conditions imposed on the production function F(K,L) for model MM imply the admissibility conditions imposed on the per-capita production function f(k) = F(K/L,1) for model PCMM.
PART C [4 Points Total]:

C.1 [1 Point]: Referring to the definition of a basic causal system (BCS) in Section 4.6 of ref. [1], derive the BCS for model PCMM. Be sure to justify with care why your derived model is indeed the BCS for model PCMM.

C.2 [3 Points]: Using the BCS you derived in Part C.1, establish carefully that, for any given admissible specification for \((\tau, \delta, g, f(\cdot))\), model PCMM has a unique admissible stationary solution \(k^*(\tau, \delta, g, f(\cdot))\) for \(k\) that is a well-defined function of the three exogenous variables \((\tau, \delta, g)\). Justify your assertions carefully, and illustrate your results using a carefully labeled and carefully explained figure called Figure 1.

PART D [4 Points]:

Using refs. [2] and [3], establish carefully that, for any given admissible specification for \((\tau, \delta, g, f(\cdot))\), the unique admissible stationary solution \(k^*(\tau, \delta, g, f(\cdot))\) you identified in Part C.2 is globally stable relative to the set of all admissible \((\tau, \delta, g, f(\cdot))\)-conditional solutions for model PCMM. Justify your assertions carefully, and illustrate your results using a carefully labeled and carefully explained figure called Figure 2.

PART E [3 Points Total]:

E.1 [2 Points]: Using your findings in Parts A through D above, explain carefully the degree to which government in model PCMM is able to control the level of \(k\) and the growth rate of \(k\) (i.e., \(D_+k/k\)) in the long run through admissible settings of the tax rate \(\tau\) at the initial time 0. Use one or more carefully labeled and carefully explained figures to illustrate your results.

E.2 [1 Point]: Is there any admissible setting \(\tau^*\) for the tax rate \(\tau\) in model PCMM that could be said to be an “optimal” tax rate in some economically meaningful sense? Explain carefully, using one or more figures to illustrate your claims.
MM: THE MODEL FOR EXERCISE 4 IN LEVEL FORM

Model MM Equations: For each time $t \geq 0$:

\begin{align*}
  Y(t) &= F(K(t), L(t)) \\  D_+ L(t) &= g \cdot L(t) \\  D_+ K(t) &= \tau Y(t) - \delta K(t) \\  C(t) &= [1 - \tau][Y(t)]
\end{align*}

(1) (2) (3) (4)

Model MM Classification of Variables:

*Time-t Endogenous Variables* ($t \geq 0$): $Y(t)$ (real output), $C(t)$ (real consumption), $D_+ L(t)$, $D_+ K(t)$

*Time-t Predetermined (State) Variables* ($t > 0$):

\begin{align*}
  K(t) &= \int_0^t D_+ K(r) dr + K(0) \\  L(t) &= \int_0^t D_+ L(r) dr + L(0)
\end{align*}

Admissible Exogenous Variables and Functional Forms:

Initial capital stock $K(0)$, initial labor force $L(0)$, labor force growth rate $g$, depreciation rate $\delta$, and government income tax rate $\tau$ satisfying the conditions $0 < K(0)$, $0 < L(0)$, $0 < g$, $0 < \delta < 1$, and $0 < \tau < 1$, plus a production function $F(K, L)$ that satisfies the *Standard Neoclassical Production Function Assumptions in Level Form*, as presented in Section 2 of Ref. [3].

PCMM: THE MODEL FOR EXERCISE 4 IN PER-CAPITA FORM

Model PCMM Equations: For each time $t \geq 0$,

\begin{align*}
  D_+ k(t) &= \tau \cdot f(k(t)) - [\delta + g] \cdot k(t) \\  c(t) &= [1 - \tau]f(k(t))
\end{align*}

(5) (6)

Model PCMM Classification of Variables:

*Time-t Endogenous Variables* ($t \geq 0$): $D_+ k(t)$, $c(t)$

*Time-t Predetermined (State) Variable* ($t > 0$): $k(t) = \int_0^t D_+ k(r) dr + k(0)$

Admissible Exogenous Variables and Functional Forms:

A($0)$, $g$, $\delta$, and $\tau$ that satisfy the conditions $0 < k(0) \equiv K(0)/L(0)$, $0 < g$, $0 < \delta < 1$, and $0 < \tau < 1$, plus a production function $f(k) \equiv F(k, 1)$ that satisfies the *Standard Neoclassical Production Function Assumptions in Per-Capita Form*, as derived in Section 3 of Ref. [3].