EXERCISE 8: INDIVIDUAL (17 Points Total)   **L. Tesfatsion**
DUE: Friday, November 10, 12:10pm   **Econ 502/Fall 2017**

**CAUTION: Late assignments will not be accepted – no exceptions.

Introductory Exercise on Rational Expectations

Background Materials:
   [1] ** Introduction to Rational Expectations (Syllabus Section IV.A)

Consider an economy described in partially reduced form by the following **Model M**:

Model M Equations: For each time period $t = 0, \pm 1, \pm 2, \ldots$:

(M1) $y_t = a^0 + a^1[p_t - {}_{t-1}p_t^e] + u_t$ Aggregate Supply Curve

(M2) $y_t = m_t - p_t + v_t$ Aggregate Demand Curve

(M3) $m_t = m^0 - g^0 v_{t-1}$ Monetary Policy Rule

Model M Classification of Variables:

Period-t Endogenous Variables:

   y_t = log of real GDP; m_t = log of the nominal money supply; p_t = log of the general price level.

Period-t Predetermined Variables:

   ${}_{t-1}p_t^e$ = subjective expectation of the period-t price level p_t formed by a representative private agent in the economy at the end of period t-1, conditional on his information set I_{t-1} at the end of period t-1.

Admissible Exogenous Variables:

   a^0 = log of potential real GDP, a^1 = adjustment parameter, and m^0, g^0 = government policy-rule parameters, with $a^0, a^1, m^0, g^0 > 0$; the stochastic process **u** consisting of serially-independent mean-zero supply shock terms u_t with stationary finite variance σ_u^2, $t = 0, \pm 1, \pm 2, \ldots$; and the stochastic process **v** consisting of serially-independent mean-zero velocity shock terms v_t with stationary finite variance σ_v^2, $t = 0, \pm 1, \pm 2, \ldots$. Assume, also, that **u** and **v** are *independent* stochastic processes (i.e., independent of each other).

   IMPORTANT NOTE: Assume the Model M modeler learns the values of all past realized endogenous variables and all past realized stochastic exogenous variables at the end of each period t-1.

Part A [2 Points]: Explain with care what additional assumptions must be made regarding the expectation $_{t-1}p_t^e$ formed by the representative private agent at the end of each period t-1 in order for this expectation to satisfy the definition of a "strong-form rational expectation" $E[p_t \mid I_{t-1}]$ in the sense of Ref. [1]. Be sure to include in this discussion a careful explanation of the types of information that must be contained in I_{t-1}.

Part B [6 Points]: Suppose the additional assumptions you made in Part A hold for Model M, and the expectation $_{t-1}p_t^e$ formed by the representative private agent at the end of each period t-1 is a strong-form rational expectation $E[p_t \mid I_{t-1}]$. Give a step-by-step derivation of the analytical form of this strong-form rational expectation, carefully justifying each step. Show, in particular, that this strong-form rational expectation is given by

$$E[p_t \mid I_{t-1}] \;=\; m^0 - g^0 v_{t-1} - a^0 \tag{1}$$

Part C [3 Points]: Suppose the additional assumptions you made in Part A hold for Model M, and $_{t-1}p_t^e$ takes the strong-form rational expectations form (1) for each period t. Carefully derive the following three analytical period-t Model M solutions (2) through (4) for y_t, m_t, and p_t. Show your work, and justify *each* step in your derivations with care.

$$
\begin{aligned}
y_t &= a^0 + \frac{a^1}{1+a^1} \cdot v_t \;+\; \frac{1}{1+a^1} \cdot u_t \tag{2}\\
m_t &= m^0 - g^0 v_{t-1} \tag{3}\\
p_t &= \frac{v_t - u_t}{1+a^1} + m^0 - g^0 v_{t-1} - a^0 \tag{4}
\end{aligned}
$$

Part D [6 Points]: Suppose the additional assumptions you made in Part A hold for the Model M economy, and $_{t-1}p_t^e$ takes the strong-form rational expectations form (1) for each period t. To what extent is the government able to influence (i) **realized values**, (ii) **expected values**, and (iii) **variances** for real GDP y_t, the money supply m_t, and the price level p_t in each period t for the Model M economy through the settings of its policy-rule parameters m^0 and g^0? Explain carefully, being sure to justify all of your assertions.