**CAUTION:** Late assignments will not be accepted – no exceptions.

**Introductory Exercise on Rational Expectations**

**Background Materials:**

[1] **Introduction to Rational Expectations (Syllabus Section IV.A)**

Consider an economy described in partially reduced form by the following **Model M**:

**Model M Equations:** For each time period \( t = 0, \pm 1, \pm 2, \ldots \):

\[
(M1) \quad y_t = a^0 + a^1 [p_t - \Delta p_{t-1}] + u_t \quad \text{Aggregate Supply Curve}
\]

\[
(M2) \quad y_t = m_t - p_t + v_t \quad \text{Aggregate Demand Curve}
\]

\[
(M3) \quad m_t = m^0 - g^0 v_{t-1} \quad \text{Monetary Policy Rule}
\]

**Model M Classification of Variables:**

**Period-t Endogenous Variables:**

\( y_t = \log \text{ of real GDP}; \quad m_t = \log \text{ of the nominal money supply}; \quad p_t = \log \text{ of the general price level.} \)

**Period-t Predetermined Variables:**

\( \Delta p_{t-1} = \text{subjective expectation of the period-}t \text{ price level } p_t \text{ formed by a representative private agent in the economy at the end of period } t-1, \text{ conditional on his information set } I_{t-1} \text{ at the end of period } t-1. \)

**Admissible Exogenous Variables:**

\( a^0 = \log \text{ of potential real GDP}; \quad a^1 = \text{adjustment parameter, and } m^0, g^0 = \text{government policy-rule parameters, with } a^0, a^1, m^0, g^0 > 0; \text{ the stochastic process } u \text{ consisting of serially-independent mean-zero supply shock terms } u_t \text{ with stationary finite variance } \sigma^2_u, \quad t = 0, \pm 1, \pm 2, \ldots; \text{ and the stochastic process } v \text{ consisting of serially-independent mean-zero velocity shock terms } v_t \text{ with stationary finite variance } \sigma^2_v, \quad t = 0, \pm 1, \pm 2, \ldots. \text{ Assume, also, that } u \text{ and } v \text{ are independent stochastic processes (i.e., independent of each other).} \)

**IMPORTANT NOTE:** Assume the Model M modeler learns the values of all past realized endogenous variables and all past realized stochastic exogenous variables at the end of each period \( t-1 \).
Part A [2 Points]: Explain with care what additional assumptions must be made regarding the expectation $t-1p_e^t$ formed by the representative private agent at the end of each period $t-1$ in order for this expectation to satisfy the definition of a “strong-form rational expectation” $E[p_t | I_{t-1}]$ in the sense of Ref. [1]. Be sure to include in this discussion a careful explanation of the types of information that must be contained in $I_{t-1}$.

Part B [6 Points]: Suppose the additional assumptions you made in Part A hold for Model M, and the expectation $t-1p_e^t$ formed by the representative private agent at the end of each period $t-1$ is a strong-form rational expectation $E[p_t | I_{t-1}]$. Give a step-by-step derivation of the analytical form of this strong-form rational expectation, carefully justifying each step. Show, in particular, that this strong-form rational expectation is given by

$$ E[p_t | I_{t-1}] = m^0 - g^0 v_{t-1} - a^0 $$

(1)

Part C [3 Points]: Suppose the additional assumptions you made in Part A hold for Model M, and $t-1p_e^t$ takes the strong-form rational expectations form (1) for each period $t$. Carefully derive the following three analytical period-$t$ Model M solutions (2) through (4) for $y_t$, $m_t$, and $p_t$. Show your work, and justify each step in your derivations with care.

$$ y_t = a^0 + \frac{a^1}{1+a^1} \cdot v_t + \frac{1}{1+a^1} \cdot u_t $$

(2)

$$ m_t = m^0 - g^0 v_{t-1} $$

(3)

$$ p_t = \frac{v_t - u_t}{1+a^1} + m^0 - g^0 v_{t-1} - a^0 $$

(4)

Part D [6 Points]: Suppose the additional assumptions you made in Part A hold for the Model M economy, and $t-1p_e^t$ takes the strong-form rational expectations form (1) for each period $t$. To what extent is the government able to influence (i) realized values, (ii) expected values, and (iii) variances for real GDP $y_t$, the money supply $m_t$, and the price level $p_t$ in each period $t$ for the Model M economy through the settings of its policy-rule parameters $m^0$ and $g^0$? Explain carefully, being sure to justify all of your assertions.