EXERCISE 9: INDIVIDUAL (10 Points Total)  
L. Tesfatsion  
DUE: 11am, Tuesday, November 17, 2015  
Econ 502/Fall 2015

**CAUTION: Please note special Tuesday due date. Late assignments will not be accepted – no exceptions.**

**Note 1:** Please make an extra copy of your exercise to bring to class on the due date for use in class discussion after you turn in your exercise.

**Note 2:** Students are permitted to work together in study groups on this exercise, but each student is asked to separately prepare and turn in their exercise answer. The techniques covered in this exercise are essential tools for subsequent parts of the course, so “free riding” on other people’s work should definitely be avoided!

**Solution of Simple Linear Rational Expectations Models**

**Background Materials:**

- Packet 23: Introduction to Rational Expectations
- Packet 24: Lucas Critique, Time Inconsistency, and Related Issues

Consider the following model of an economy over times $t \geq 1$:

**Model Equations:**

\[
\begin{align*}
(1)^{\ast} & \quad y_t = y_t^* + ap_t + bE_{t-1}p_t ; \\
(2)^{\ast} & \quad m_t = ky_t + p_t + \epsilon_t ; \\
(3)^{\ast} & \quad E_t p_{t+1} = E[p_{t+1}|I_t] ,
\end{align*}
\]

where:

- $y_t^*$ notes the log of potential real GDP in period $t$;
- $y_t$ denotes the log of actual real GDP in period $t$;
- $p_t$ denotes the log of the general price level in period $t$;
- $m_t$ denotes the log of the nominal money supply in period $t$;
- $E_t p_{t+1}$ denotes the subjective forward-looking expectation of a representative agent in period $t$ regarding the price level in period $t + 1$.
- $I_t$ denotes the set of information available to the representative agent at the beginning of period $t + 1$ (i.e., at the end of period $t$) whose contents are assumed to be consistent with strong-form RE. (Important Note: For the purposes of this exercise, assume the modeler is able to observe all past realized values for $y_t$ and $p_t$ as time $t$ proceeds.)
Classification of Model Variables:

Time-t Endogenous Variables ($t \geq 1$):

\[ y_t, \; p_t, \; E_t p_{t+1} \]

Time-t Predetermined Variable ($t > 1$):

\[ E_{t-1} p_t \]

Exogenous Variables and Admissibility Conditions:

The deterministic exogenous variables are the exogenous constants $a, b, k$, \{\( y_t^* | t = 1, 2, \ldots \}\}, the money supply settings \{\( m_t | t = 1, 2, \ldots \)\}, and \( E_0 p_1 \). These deterministic exogenous variables satisfy the following admissibility conditions: $k > 0, a > 0, [a + b] \geq 0$.

The stochastic exogenous variables are the random error terms \{\( \epsilon_t | t = 1, 2, \ldots \)\}. These random error terms satisfy the following admissibility condition: they constitute a serially independent process satisfying $E[\epsilon_t | I_{t-1}] = 0$ for all $t \geq 1$.

The model is incomplete as it stands, in that the “true conditional expectation” on the right hand side of model equation (3)* needs to be determined in a manner consistent with strong-form rational expectations (RE), and this true conditional expectation — backdated to period $t-1$ — needs to be substituted in for the subjective expectation $E_{t-1} p_t$ appearing in model equation (1)* for each $t \geq 1$. This includes making sure that the exogenous initial subjective expectation $E_0 p_1$ is set in conformity with strong-form RE.

This exercise asks you to solve for the needed strong-form RE in (3)* and to interpret the implications of assuming strong-form RE within the model.

Part A (2 Points): Specify carefully and precisely the information that must be included in the information set $I_{t-1}$ for each $t \geq 1$ in order for this information set to be in accordance with the definition of strong-form RE for the problem at hand.

Part B (6 Points): Using your results from Part A, determine explicit analytical expressions for the strong-form RE for both $p_t$ and $y_t$, i.e., analytical expressions that represent $E[p_t | I_{t-1}]$ and $E[y_t | I_{t-1}]$ as functions solely of the information in $I_{t-1}$. Be sure to show your derivations step by step, and be sure to justify carefully each of these steps.

Part C (2 Points): Do your results from Part B support the assertion that government monetary policy is necessarily ineffective under strong-form RE, in the sense that government does not have the ability through choice of its monetary policy variable $m_t$ in each period $t \geq 1$ to make $E[y_t | I_{t-1}]$ deviate systematically from $y_t^*$? For example, can government maintain $E[y_t | I_{t-1}]$ at some target level $\hat{y}_t > y_t^*$ in each period $t$ by an appropriate choice of its monetary policy variables $m_t$? Explain carefully why or why not.