Iowa State University
Department of Economics
Econ 509
Summer 2008

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Applied Numerical Methods in Economics

Homework #4

Electronic answers due on 07/09/08 (Wednesday) at 1:00 PM

1. Computing Discounted Utility with the Gauss-Legendre Quadrature

   If the time t consumption of an agent can be described by $c(t) = 1 + t/5 - 7(t/50)^2$, and $t \in [0, 50]$, the discounted utility U is:

$$U = \int_0^{50} \exp(-\rho t) u(c(t)) dt$$

   where $u(c) = (1-\gamma)^{-1} c^{1-\gamma}$. Assume that $\gamma = 3$ and that $\rho = 0.05$

   Compute numerically the value U programming a self-contained script m-file that applies the the Gauss-Legendre quadrature with 7 nodes. The 7 nodes and weights for that quadrature are detailed below (and are also available at the course website).

Nodes	Weights
-0.9491079123	0.1294849662
-0.7415311856	0.2797053915
-0.4058451514	0.3818300505
0.0000000000	0.4179591837
0.4058451514	0.3818300505
0.7415311856	0.2797053915
0.9491079123	0.1294849662

   Check your result by using the function `qnwlege` of the CompEcon library.

2. Portfolio Selection with Gauss-Hermite Quadratures

   The most basic portfolio optimization problem can be described as:

$$\max_{\{\omega_i\}_{i=1}^{n}} \mathrm{E}\left[= u\left(\sum_{i=1}^{n} \omega_i Z_i\right)\right]$$

subject to:

$$\sum_{i=1}^{n} p_i(\omega_i - e_i)$$

where e_i is the initial endowment of asset i, and p_i is the price of that asset. The ω's are assets shares in the portfolio, so that $\sum_i \omega_i = W$.

We are going to specialize this problem by setting $n = 2$, i.e., by working with two assets, one of which (asset 1) is a safe asset with a known rate of return $Z_1 = R = 1.1$. The rate of returns of asset 2, Z_2, is uncertain and the log of the future value of the asset is $Z_2 \sim N(\mu, \sigma^2)$. The assets' prices are assumed to be $p_1 = p_2 = 1$. Furthermore, we assume that $u(c) = c^{1+\gamma}/(1+\gamma)$, $e_1 = W = 2$ and $e_2 = 0$. Therefore, we can write $\omega_1 = W - \omega_2$ and let $\omega = \omega_2$ be the choice variable.

With the above simplifications, the problem reduces to the following unconstrained problem:

$$\max_{\omega} \mathrm{E}\left[u\left(R(W-\omega) + \omega \exp(Z_2)\right)\right]$$

This objective function and its derivatives can be approximated numerically. For that end, assume $(\mu, \sigma^2) = (0.2, 0.3)$.

(a) Obtain the expected value for the portfolios arising from considering 10 evenly spaced values of ω between 0 and W, and the following five values of γ: -0.5, -1.1, -2, -5, and -10.

(b) Use Newton's optimization method to maximize $U(\omega)$ and use Gauss-Hermite integration to evaluate $U(\omega)$. Use 5 and 7 Gauss-Hermite nodes and set $\gamma = -5$.

3. Write a script m-file to compute the derivative of the function $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ and report the percentage relative errors of estimation that arise when you use "reasonable" values of h and the following formulae:

(a) The backward- and forward-finite difference formulae discussed in class.

(b) The two-sided formula discussed in class.

(c) The following forward- and backward-finite difference, respectively FD and BD, that incorporate more terms of the Taylor series discussed in class:

$$\text{FD: } f'(x) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} \qquad \text{BD: } f'(x) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$$

(d) The following centered-finite difference that incorporates more terms of the Taylor series discussed in class:

$$f'(x) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

Consider a range of values of x to report your results. A graph should help to visualize the results.

4. MALTHUS EQUATION AND THE SOLUTION TO SIMPLE DIFFERENTIAL EQUATIONS

   (a) The assumption of a constant rate of population growth is called the Malthus' law and it leads to a simple differential equation: $dp(t)/dt = r\,p(t)$, where $p(t)$ is the population size at time t and r is the constant population growth rate. The solution to the initial value problem posed by the Malthus' equation is $p(t) = p(0)\exp(rt)$.

   Compare this solution with the numerical solution that arises after applying the Euler's Method discussed in class and the CompEcon function `rk4`. Use $\rho = 0.02$, $h = 0.01$, and time periods $t = 0 + i\,h$, with $i = 1, ..., h \times T$ with $T = 50$. Use a graph for each solution method to show the difference between the analytical and numerical solutions.

   (b) Now, instead of a constant population growth, assume that $dp(t)/dt = p(t)(a - bp(t))$, and set $T = 2$, $a = 4.5$, and $b = 4$. This is called the Logistic Model. Plot the solution $p(t)$ that you find to this differential equation using the function `rk4` of the CompEcon library.

   (c) Verify that the solution to $dy(t)/dt = y(t)^2 + y(t)/t$ is $y(t) = 2t/(c - t^2)$. What is the value of c?

5. THE APPROXIMATION TOOL KIT IN THE COMPECON LIBRARY

   Choose a function of your interest to be approximated using the tool kit available at the CompEcon library. Write a script m-file where you can show how to use the following routines available in that library: `fundefn`, `funfitf`, `funfitxy`, `funeval`, and `funbas`. Use these routines with at least three different family of functions.

6. COURNOT PROBLEM AND THE COLLOCATION STRATEGY

   In the Cournot problem discussed in class, the effective supply function was characterized by the following functional equation:

$$p + \frac{S(p)}{D'(p)} - MC(S(p)) = 0$$

   where p is the price, S the supply function, D the demand function, and MC the marginal cost function.

   Assume that the demand function is $D(p) = p^{-1.5}$ and that the marginal cost is constant, i.e., $MC(q) = c = 1$. Use a collocation strategy like the one discussed in class to obtain an approximation for the function $S(p)$. Plot that function. Indicate the equilibrium price when there are 4 firms in the market.