FEMALE LABOR SUPPLY WITH A DISCONTINUOUS, NONCONVEX BUDGET CONSTRAINT: INCORPORATION OF A PART-TIME/FULL-TIME WAGE DIFFERENTIAL
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Abstract—This paper incorporates the well-documented part-time/full-time wage differential into an empirical labor supply model with both a heterogeneity- and a random-error term and estimates that model for women in the United States. Incorporation of the part-time/full-time wage differential results in a unique discontinuous, nonconvex budget set, and the consideration of estimation procedures previously unconfonnted in the nonlinear budget constraint literature. The full structural representation of the budget constraint is shown to “fit” better than the alternative models estimated, and to yield a predicted hours distribution representative of actual hours.

I. Introduction

The bulk of the literature relating to part-time employment concerns itself with documenting the pay differential between full-time and part-time workers. In this literature, full-time workers are found to earn anywhere from 3% to 30% more per hour than part-time workers. The purpose of this paper is to examine the role the part-time/full-time wage differential plays in determining the labor supply of women by structurally incorporating that wage differential into the budget set that decision makers face. The process will also allow us to learn something about introducing a discontinuity into the previously developed nonlinear budget set estimation problem. This knowledge can be applied to the more general analysis of tied wage–hours contracts.

Estimating the typical labor supply function assumes a degree of flexibility regarding hours choice that may not exist. Job offers that stipulate both a wage and hours of employment (take it or leave it) do not coincide with the usual assumption that workers choose their level of hours based on some optimizing criterion. Kiefer (1988) explores the theoretical implications of tied wage–hours offers. Tummers and Woittiez (1991), Lundberg (1985), Altonji and Paxson (1988), and Moffitt (1984b) present empirical evidence indicating that wages are not truly exogenous—wages are tied to hours of employment. And the degree to which the assumption of exogenous wages (unconstrained optimization) biases labor supply estimates is examined by Kahn and Lang (1991). In addition, the endogeneity of wages results in a budget constraint that deviates considerably from the linear, continuous version that is often assumed in the estimation of labor supply.

Unlike the formal contracts that motivate much of the literature on tied wages and hours, the part-time/full-time wage differential arises informally in the labor market as a result of quasi-fixed labor costs (Oi (1962)). The incidence of a higher wage being paid to full-time (high-hours) workers and a lower wage being paid to part-time (low-hours) workers provides a natural setting in which to examine the importance of tied wage–hours offers in the estimation of labor supply.

The next section introduces the theoretical nature of the part-time/full-time wage differential. Section III details various empirical specifications of the supply of labor. The data used and preliminary empirical steps are discussed in section IV. The labor supply estimation results are presented in section V, followed by concluding remarks in section VI.

II. Theoretical Nature of the Part-Time/Full-Time Wage Differential

A. Budget Set

Figure 1 illustrates the budget set a worker faces when both full-time and part-time work is available. Given that full-time workers earn more than part-time workers with identical, observable characteristics (Blank (1990) and Ehrenberg et al. (1988)), there is a discontinuity at point $H^*$ (the point at which full-time employment is defined).

The budget constraint is nonconvex and discontinuous at $H^*$ because, unlike in the case of overtime, if a person has a full-time job, he or she will earn $W_F$ for each hour worked (not $W_P$ until $H^*$ and then $W_F$). In other words, persons working 40 hours per week do not get paid the part-time wage for the first 35 hours and then the full-time wage for the last 5; instead they get the full-time wage for all 40 hours.

In addition to affecting structural labor supply estimates, the discontinuity raises the issue of overemployment, where an individual may elect to supply $H^*$ number of hours, given the shape of the budget set, but if the higher wage were available, would prefer to work fewer hours. (The indifference curve depicted intersects the point of discontinuity, dips below the dotted segment, and lies everywhere above the part-time segment.) If this is the case for a considerable number of observations, the implicit assumption of observed hours resulting from the first-order conditions of a utility-maximizing problem is incorrect. The estimation of labor supply in this paper allows for deviations of observed hours from even the desired hours segment by structurally incorporating the discontinuity and nonconvexity introduced by the...
presence of a part-time/full-time wage differential. We account for the full budget set when estimating the model.

B. Incorporating the Nonlinear Budget Constraint

Recent years have seen an attempt to incorporate more realistic depictions of the budget set, distorted by a variety of tax laws and welfare schemes. This paper joins that literature in a slightly different vein. Whereas most of the distortions usually considered are introduced by some governmental policy or institutional mechanism, the nonlinearity of the budget set in this paper results from the operation of the market and the profit-maximizing behavior of employers. The idea of labor as a quasi-fixed input was introduced by Oi (1962). As such, labor generates a quasi-fixed cost, which implies that lower wages will be paid to low-hours workers and higher wages to high-hours workers. Workers must put in some minimum number of hours for the firm to recoup the fixed (hiring and training) cost associated with their employment. Even quasi-fixed hiring and training costs associated with the number of employees rather than hours worked by employees encourages firms to hire workers for longer hours at higher wage rates (Lewis (1969)). Such explanations are consistent with the evidence offered earlier, documenting tied wage–hours offers and the part-time/full-time wage differential, which leads to the discontinuous, nonconvex budget constraint in this paper.

Recent estimates of labor supply have found that incorporating a nonlinear budget constraint changes the calculated wage and income labor supply elasticities substantially. Although previous research has primarily focused on nonmarket-generated distortions to the budget set, we expect that the structural modeling of the tied wage–hours phenomenon, with explicit incorporation of the part-time/full-time wage differential, will also lead to different wage and income labor supply elasticities. The shape of the budget constraint depicted in figure 1 illustrates how an estimation methodology that assumes a continuous budget constraint would tend to overestimate the impact of the wage on labor supply decisions when the worker faces a part-time/full-time wage differential (or some other similar tied wage–hours package). Conceptually, a specification that assumes a continuous budget constraint forces the wage coefficient to capture the large wage change that occurs as hours move from part-time to full-time. The problem with linear, continuous labor supply specifications is that they do not take into account the endogeneity of the wage faced by the individual. The wage that affects a person’s labor supply decision is different depending on the labor supply decision (full-time versus part-time) itself. The simplest methodology that has been employed to account for this endogeneity is an instrumental-variables (IV) approach, which is essentially an ordinary least-squares (OLS) specification that controls for the full-time/part-time segment choice. While the IV approach does account for the endogeneity of the wage (segment choice), it fails to account explicitly for individuals at the point of discontinuity (see Moffitt (1990)). This is a problem from which the dual-error-term model detailed below does not suffer.

III. Empirical Specification

This section introduces the dual-error-term model, which incorporates the part-time/full-time wage differential into a labor supply model with both a heterogeneity- and a random-error term. Variations of this dual-error-term structure which include either a heterogeneity- or a random-error term have been estimated in the literature in different contexts, but are less flexible versions of the model estimated here. One error term is added to account for heterogeneity within the sample. It allows us to account for two observationally equivalent people working a different number of hours. The second error term is the standard random error; it allows us to observe a person working a different number of hours than may be optimal. The dual-error-term structure has the advantage of integrating theory and econometrics since the budget set is used to derive the likelihood function.

Using a modified form of Moffitt’s (1984a) nonlinear specification, the structural estimation of labor supply, which incorporates the part-time/full-time wage differential, can be outlined as follows (refer to figure 2). Here $P$ is used

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3 See also Pencavel (1986, p. 36).

4 See, for example, Moffitt (1984b), Pencavel (1986), and Hausman (1985a).
to define items related to the part-time segment and $F$ to define items related to the full-time segment. The wage is denoted by $W$, nonlabor income by $Y$; $h$ denotes the number of hours worked per week (amount of labor supplied) and $H^*$ the number of hours per week at which a person is considered full-time employed. Also, define the following, where $i = P, F^*$:

- $h^* = g(W_i, Y) + \epsilon_h$, desired demand for leisure
- $h = h^* + \epsilon_r = g(W_i, Y) + \epsilon_r + \epsilon_h$, observed demand
- $\epsilon_h = \text{heterogeneity error term}$
- $\epsilon_r = \text{random-error term}$

Define $k(i, h) = h - g(W_i, Y)$ as the value of $\epsilon_h$ that makes the desired hours in sector $i$ equal to the observed hours,

$$k(i, h) = h^* + \epsilon_r - g(W_i, Y) = g(W_i, Y) + \epsilon_h + \epsilon_r - g(W_i, Y) = \epsilon_r + \epsilon_h$$

setting this equal to

$$\epsilon_h \Rightarrow \epsilon_r = 0 \ (\text{optimization error} = 0).$$

Define $m(P, F)$ as the value of $\epsilon_h$ that equilibrates utility on the part-time and full-time segments. This is implicitly defined by

$$U[H^*, W, H^* + Y, m(P, F)] = V[W, Y, m(P, F)] \quad (1)$$

where $U[\cdot]$ is the direct utility function and $V[\cdot]$ is the indirect utility function.

This differs from Moffitt (1984a) in the sense that the direct utility at $H^*$ must be equated with the indirect utility obtained at the optimum on the part-time segment, because the evaluation of utility at $H^*$ is not at a point of tangency on the full-time segment (which is implied by the indirect utility function). A closed-form solution for $m(P, F)$ does not exist, so it must be solved for numerically. The model is estimated for a discontinuity in the budget constraint, which occurs at 36 hours per week. The likelihood function for the dual-error-term model is presented in the appendix.

For estimation one must specify a functional form for $g(\cdot)$ and then determine $m(\cdot)$. The linear labor supply function is chosen for $g(\cdot)$, since closed-form solutions for both the indirect and the direct utility functions can be found from this specification. The labor supply function is given as

$$g(W_i, Y) = Z\alpha + \beta W_i + \delta Y. \quad (2)$$

The indirect and direct utility functions that correspond to the linear labor supply equation are given by

$$V(W_i, Y, \epsilon_h) = e^{\delta W_i} \left( Y + \frac{\beta}{\delta} W_i - \frac{\beta}{\delta^2} + \frac{s}{\delta} \right) \quad (3)$$

and

$$U(h, C, \epsilon_h) = e^{-\left[1 + \delta(C + \delta s - \delta b) \right]} \left( \frac{-b}{\delta} \right) \quad (4)$$

where $b = \beta/\delta$, $s = Z\alpha + \epsilon_h$, and $\delta = s/\delta - \beta/\delta^2$.

The statistical software package GAUSS is used to maximize the likelihood function. A standard Heckman selection model, a tobit model, and an IV model are also estimated for comparison purposes. All three models assume a linear budget constraint and incorporate the standard random-error term only. The IV model, however, does account for the endogeneity of the wage. A priori, we would expect the dual-error-term model to perform better in replicating the hours distribution since it is able to account for and assign variations in hours that come from both optimization errors and unmeasured differences across individuals (heterogeneity).

The dual-error-term model was estimated for discontinuities at 30, 35, and 36 hours. The discontinuity at 36 hours was chosen as preferred because it yields the greatest likelihood function value. See Hotchkiss (1991) and Averett and Hotchkiss (1996) for discussions of this grid-search method for determining the most likely hours at which the discontinuity occurs. Discontinuities at 37, 38, and 40 hours were also considered, but we were not able to obtain convergence for discontinuities greater than 36. The similarity of parameter estimates across the models estimated makes us confident that if a discontinuity at some hours other than 36 was preferred, the results presented in this paper would not differ appreciably. In addition, a discontinuity between 35 and 40 hours is consistent with discontinuities identified by Hotchkiss (1991) and Averett and Hotchkiss (1996).

The analytic gradients for this model are available from the authors upon request.
IV. Data and Preliminary Empirical Steps

A. Data

The structural labor supply model is estimated for noninstitutional females ages 20 to 60 years old using cross-sectional data constructed from the March 1989 Current Population Survey (CPS). Women in the outgoing rotation groups only were selected because of detailed employment information asked only of these groups. Individuals are deleted if their sources of nonwage income (necessarily from the previous 12 months) include unemployment insurance, public assistance (or welfare), disability income, social security income, or supplemental security income. These income sources have been shown to distort labor supply decisions (Danziger et al. (1981) and Killingsworth (1983)). Job information refers to the respondent’s current or most recent job. Table 1 contains the means for the sample constructed from the CPS.

Nonwage income is calculated as total family income minus the individual’s earnings. This means that earnings of all other family members are treated as nonwage income by the individual. Hourly wage is the wage that is reported for hourly wage earners, and is constructed (using annual earnings and usual hours per week) for salaried workers.

The CPS is a nationally representative sample and the means reflect expected national averages for women. We include women between the ages of 20 and 60 only to be able to compare the results with other studies of women in their prime working years. This restriction still allows for a substantial number of part-time workers. The distribution of the sample across hours of work per week is found in figure 3 (which also contains the predicted hours distributions, discussed in section V.C). We observe the usual clumping of workers at 20, 35, and 40 hours per week.

B. Prediction of Wages

In order to estimate the structural labor supply model, we must observe a part-time and a full-time wage for every individual. This creates a problem because at most we observe only either a full-time or a part-time wage, but never both, and no wage at all if the person is not employed. To obtain a part-time and a full-time wage for everyone, selectivity-corrected wage equations are estimated for the full-time and the part-time subsamples, and the consistently estimated parameters are then used to calculate a predicted full-time and part-time wage for each observation: these are the $W_F$ and $W_P$ in the likelihood specification. This step is not always necessary in the usual nonlinear budget set formulation since the slope of each segment can be calculated from observable information (i.e., the actual wage and

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The data were obtained from the Interuniversity Consortium for Political and Social Research (see U.S. Dept. of Commerce (1990)).

Individuals with negative nonwage income are also deleted to avoid additional (difficult to support theoretically) complications to the budget set.

We did not exclude women who might be deemed (imprecisely, at best) eligible for these income sources but chose not to receive them. Consequently, as pointed out by a referee, we may have oversampled those with a fairly strong taste for work. Our results, therefore, are conditional on not choosing to participate in a welfare program.

The table also reflects the elimination of 40 women whose predicted part-time wages were greater than their predicted full-time wages. See the following section for discussion of this point.

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TABLE 1.—Means and Standard Deviations from the March 1989 CPS Sample Used for Estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All respondents (8274 individuals)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>37.29</td>
<td>11.02</td>
</tr>
<tr>
<td>Education</td>
<td>13.15</td>
<td>2.45</td>
</tr>
<tr>
<td>Children less than 6 years</td>
<td>0.31</td>
<td>0.64</td>
</tr>
<tr>
<td>Nonwage income ($000)</td>
<td>27.60</td>
<td>25.64</td>
</tr>
<tr>
<td>Single = 1</td>
<td>0.33</td>
<td>0.47</td>
</tr>
<tr>
<td>Black = 1</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>West = 1</td>
<td>0.25</td>
<td>0.44</td>
</tr>
<tr>
<td>South = 1</td>
<td>0.32</td>
<td>0.47</td>
</tr>
<tr>
<td>Central city = 1</td>
<td>0.21</td>
<td>0.41</td>
</tr>
<tr>
<td>Enrolled in school = 1</td>
<td>0.04</td>
<td>0.20</td>
</tr>
<tr>
<td>Workers (5577 individuals)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual hours worked per week</td>
<td>36.39</td>
<td>10.03</td>
</tr>
<tr>
<td>Hourly wage ($)</td>
<td>8.88</td>
<td>5.04</td>
</tr>
</tbody>
</table>

Number working less than 36 hours per week = 1686 (30% of workers)
the tax rate). In future work we hope to endogenize the wage determination fully.\(^{15}\)

In estimating the wage equations, one needs to control both for selection into the labor force and for selection into the specific sector (part-time or full-time).\(^{16}\) Since previous research has suggested that a worker’s choice of hours is determined separately from the decision to work,\(^{17}\) and due to the sequential nature of those decisions, a sequentially ordered response selectivity model is estimated to obtain a full-time and a part-time wage for everyone in the sample.\(^{18}\)

Using these parameter estimates to predict full-time and part-time wages for everyone in the sample resulted in less than 0.5% (40 in all) of the sample having a higher part-time predicted wage than the full-time predicted wage. These individuals were eliminated from the final maximum-likelihood estimation.\(^{19}\)

One concern about eliminating these individuals is that they may not be representative of the entire sample, that is, there could be some systematic reason that their part-time wage was estimated to be greater than their full-time wage. If this is the case, then eliminating them from the sample could bias our parameter estimates. In order to ensure that this is not the case, we estimated the tobit and OLS selection models including these 40 women and compared the resulting parameter estimates with those obtained in this paper. There was basically no difference in the results, giving us confidence that their exclusion also does not affect dual-error-term results.\(^{20}\) Although the structure imposed in the dual-error-term model assumes that for any given job, a full-time worker must earn more than a part-time worker, the structure does allow a full-time worker at one job to earn a lower wage than a part-time worker at another job. It is therefore possible to see a part-time worker earning a higher wage than a different full-time worker.

**C. Determination of \(m(P, F)\)**

The prediction of \(W_P\) and \(W_F\) leaves only one additional variable entering the likelihood function that is not readily obtainable or constructed from information in the data set. That variable is \(m(P, F)\), which is the value of the heterogeneity error term that equates utility on the part-time and full-time segments. Rearranging the indirect and direct utility functions that correspond to the linear labor supply function (equations (3) and (4)), replacing \(C = W_FH^* + Y\), substituting

\[
\delta = \frac{s}{\delta^2} - \frac{\beta}{\delta^2} = \frac{1}{\delta} \epsilon_h - \frac{\beta}{\delta^2}
\]

and equating the two to get an implicit function for \(\epsilon_h\), we have

\[
V(W_P, Y, \epsilon_h) = e^{\delta W_P} (y + \frac{\beta}{\delta} W_P - \frac{\beta}{\delta^2} + \frac{1}{\delta} \epsilon_h)
\]

\[
= e^{[1 + \epsilon_h \delta W_P - \delta Y^* - \alpha Z^* + b(\delta - h)]} \times \left( \frac{H^* - b}{\delta} \right)
\]

(5)

Since there is no closed-form solution for \(\epsilon_h\), it is solved for numerically using the bisection method of numerical analysis. The subroutine that solves for \(m(P, F)\), or \(\epsilon_h\), is included in the maximum-likelihood procedure and is called for each person at each iteration, since each person will have a different value of \(m(P, F)\), and that value depends on the estimated parameters, which change at each iteration.\(^{21}\)

A necessary condition for the existence of an \(\epsilon_h\) that equates utility on the full-time and part-time segments, is the satisfaction of the Slutsky condition, which ensures concavity of the utility function.\(^{22}\)

**V. Labor Supply Results**

**A. Parameter Estimates**

Three alternative empirical specifications are estimated in order to compare the results from the dual-error-term model with results generated by more common, simpler, but less complete methodologies. The first specification is a Heckman (1979) selectivity model, which involves constructing an additional regressor that reflects the worker’s probability of being observed in the labor market and then estimating the labor supply function via OLS for workers only.\(^{23}\) The tobit model (Tobin (1958)) is the second specification and

\(^{15}\) MaCurdy et al. (1990) discuss methods to endogenize the wage structurally, but to our knowledge this methodology has not yet been implemented.

\(^{16}\) Ilmakunnas and Pudney (1990) use OLS wage predictions not corrected for selectivity, making the potential for measurement error even more severe.

\(^{17}\) The presence of fixed costs is typically cited as the reason for this finding. (See Blank (1988).)

\(^{18}\) This model is discussed in some detail in Maddala (1983, pp. 49–51). A supplemental appendix containing the sequentially ordered response estimates and the final selectivity-corrected wage equations is available from the authors upon request.

\(^{19}\) We felt these were truly outliers since, contrary to what Blank (1990, p. 219) found for 1987, part-time workers in our sample have lower mean (and median) hourly wages than full-time workers in all occupations. The only exception is women in technical and related support positions (159 women). A potential source of these outliers is the recently revealed failure of the CPS to accurately account for part-time workers if they hold more than one job. It is estimated that 2% of “full-timers” are actually individuals holding dual part-time jobs (Uchitelle (1993)).

\(^{20}\) The results of this sensitivity analysis are available from the authors upon request.

\(^{21}\) The FORTRAN subroutine is available from the authors upon request.

\(^{22}\) See Triest (1990) and MaCurdy et al. (1990). Hausman (1981) does not need to impose the Slutsky condition because his heterogeneity error is part of the income term, which is then chosen to come from a truncated distribution.

\(^{23}\) A non-selectivity-corrected OLS labor supply function is estimated for workers only as well. These results do not differ substantially from the OLS selection results and are reported in a supplemental appendix available from the authors.
involves estimating a likelihood function over both workers and nonworkers, with each contributing differently to the likelihood function. Since we do not observe wages for nonworkers, imputed wages are used in the tobit estimation. The imputed wages are obtained by estimating wage equations on the workers controlling for self-selection into the labor force, then using the selectivity-corrected parameter estimates to predict a wage for everyone in the sample. Results from the probit selectivity equation and the OLS wage equation used to obtain the imputed wage used in the tobit estimation are available from the authors in a supplemental appendix. Neither the OLS selection model nor the tobit model accounts for the endogeneity of the wage.

The third alternative specification is an IV approach, which is essentially an OLS estimation that controls for each individual’s (full-time or part-time) segment choice. A selection term is constructed to reflect an individual’s probability of either working and locating on the full-time segment or working and locating on the part-time segment, depending on the individual’s observed hours. The selection term is then included as a regressor in the OLS estimation of the hours equation. (These selection terms are the same selection terms used to predict full-time and part-time wages for use in the dual-error-term estimation.) Table 2 contains the parameter estimates for the OLS selection model, tobit model, IV model, and dual-error-term model. Obtaining final labor supply estimates for each model involves a number of estimation steps, raising the issue of identification. We chose to exclude some regressors from the final labor supply equation that were included in the preliminary wage equation estimations for identification purposes. (All of the regressors from the wage equation were also included in the labor force participation equation estimated in the first step of the OLS selection model in order to represent the importance of the wage in determining labor force participation.) The excluded regressors are age squared, education squared, age times education, and region and central city dummies. While the exclusion of age squared, education squared, and age times education would arguably identify the model through their nonlinearity, we also felt it was reasonable to expect wages to vary systematically across regions and central city residence designation (see Ihlanfeldt (1989)), but that hours would not. The labor supply estimates are robust to combinations of regional and central city dummy exclusion restrictions, and the coefficients on the excluded variables are significantly different from zero. The one exception was the wage coefficient in the dual-error-term model, which became a small negative number under one of the combinations. However, even this outcome does not change the overall interpretation of the results nor the outcome of the hours predictions.

Comparing the labor supply estimates in table 2, we find that the results are relatively similar. Being single exhibits a positive and significant effect on labor supply across each specification. Nonwage income is negative and significant in the OLS selection, tobit, and dual-error-term models. The coefficient on nonwage income from the IV estimation is insignificant. The presence of children less than six years old (except for the IV model) and being enrolled in school both reduce women’s labor supply, as would be expected. The wage coefficient is positive and significant in each specification, but the coefficient in the dual-error-term model is less significant and considerably smaller than that obtained from the tobit estimation, slightly smaller than the OLS selection estimate, and slightly larger than the IV estimate. The

| Table 2.—Labor Supply Parameter Estimates |
|---|---|---|---|
| Variable | OLS Selection Model | ML Tobit Model | IV Model | Dual-Error-Term Model |
| Intercept | 3.345 | 1.850 | 2.824 | 1.433 |
| (0.160) | (0.253) | (0.086) | (0.192) |
| Age (10s) | $-0.072^a$ | $-0.393^a$ | $-0.0005$ | $-0.312^a$ |
| (0.017) | (0.034) | (0.010) | (0.027) |
| Education (10s) | $0.287^a$ | $0.913^a$ | $-0.457^a$ | $1.284^a$ |
| (0.100) | (0.260) | (0.058) | (0.192) |
| Single = 1 | $0.204^a$ | $0.663^a$ | $-0.211^b$ | $0.278^a$ |
| (0.039) | (0.070) | (0.024) | (0.064) |
| Black = 1 | $0.102^a$ | $0.331^b$ | $-0.062^a$ | $0.196^b$ |
| (0.036) | (0.094) | (0.030) | (0.089) |
| Nonwage income (0000) | $-0.072^a$ | $-0.163^b$ | $0.001$ | $-0.215^a$ |
| (0.008) | (0.013) | (0.007) | (0.008) |
| Children less than 6 years | $-0.354^a$ | $-1.001^b$ | $0.033$ | $-0.832^a$ |
| (0.046) | (0.051) | (0.029) | (0.046) |
| Enrolled in school = 1 | $-1.624^a$ | $-2.317^a$ | $-1.149^a$ | $-1.747^a$ |
| (0.101) | (0.161) | (0.101) | (0.156) |
| Wage | $0.030^a$ | $0.110^a$ | $0.016^a$ | $0.024^b$ |
| (0.004) | (0.025) | (0.003) | (0.014) |
| $\lambda$ | $0.288^a$ | — | $2.829^b$ | — |
| (0.175) | | (0.070) | |
| $\sigma$ | $0.915$ | $2.423^a$ | $0.776$ | — |
| (0.025) | | (0.070) | |
| $\sigma_r$ | — | — | — | $0.620^a$ |
| | | | | (0.005) |
| $\sigma_s$ | — | — | — | $1.945^a$ |
| | | | | (0.071) |
| Number of observations | 5577 | 8274 | 5577 | 8274 |
| Adjusted $R^2$ | 0.17 | — | 0.40 | — |
| Log likelihood | $-7415$ | $-15,339$ | $-6496$ | $-11,946$ |
| Log likelihood ($\beta = 0$) | $-7928$ | $-16,044$ | $-7928$ | $-14,894$ |

Notes: The dependent variable, hours per week, is measured in tens. Standard errors are in parentheses. Standard errors for the OLS selection model have been corrected for heteroskedasticity.

$^a$ Significance at the 1% level.

$^b$ Significance at the 5% level.

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24 An alternative would be to impute wages only for the nonworkers and use observed wages for the workers. See Killingsworth (1983, pp. 150–157) for a discussion of the various procedures available to solve the unobserved wage problem.

25 Moffitt (1990, pp. 131–133) discusses this and other versions of the IV methodology which account for the endogeneity of wages.

26 See Manski (1994) for a broad discussion of identification issues.

27 These results are available from the authors upon request.
relative magnitude of the IV and dual-error-term estimates for the wage coefficient supports the notion that endogenizing the wage is an important part of modeling the part-time/full-time wage differential. Among other shortcomings (see Moffitt (1990, p. 133)), however, the inability of the IV model to explicitly account for individuals located at the kink point still leads us to prefer the dual-error-term specification.

Education is positive and significant in each of the specifications (except the IV). A positive education coefficient suggests that there remains a significant positive impact of education on labor supply, even after controlling for education’s effect on the wage (through the estimated wage equation). This is consistent with the desire of a woman to recoup the investment she has made in her education, which translates into supplying more labor than someone who has incurred lower educational costs.

As is typical in applications of the dual-error-term model, the standard error of the heterogeneity-error term is larger than the standard error of the random-error term (see Moffitt (1986) and Triest (1990)). In the model estimated here both sources of error are highly significant, and the heterogeneity standard error is three times as large as the random error. Moffitt (1986, 1990) notes that what distinguishes the two error terms is the degree of clustering in the data around the kink points of the budget constraint. We have significant clustering at zero hours and in the full-time segment, which helps to identify the different error terms (see figure 3).

### B. Wage and Income Elasticities

Table 3 contains the wage and income elasticities from all four of the model specifications. We have followed the convention established in Killingsworth (1983) by reporting both the gross and the compensated wage elasticities. The compensated wage elasticities have the theoretically correct sign. The wage elasticities are calculated using both the predicted full-time and part-time wage rates and the actual observed wage as well. Most of the wage elasticities are significantly different than zero at the 99% significance level. (The exceptions are the gross wage elasticities for the dual-error-term model, which are significant at the 95% confidence level, and the compensated wage and income elasticities for the IV model, which are insignificantly different from zero.)

The gross wage elasticities for the dual-error-term model are considerably less than the gross wage elasticities for the tobit model, slightly smaller than the OLS selection estimate, and slightly larger than the gross wage elasticity for the IV estimation. Given that the wage elasticity is a measure of the sensitivity of labor supply to small, continuous changes in the wage, it is not surprising that the explicit incorporation of a fairly sizable, discrete wage differential along the budget constraint, through endogenizing the wage, would result in a smaller elasticity. The wage elasticities calculated for models that incorporate this wage differential take into account wage changes along the part-time/full-time segment (segment choice has been taken into account in the estimation procedure), where the variation in wages is not expected to be as great as when wage changes along the full hours spectrum are considered, as in the tobit or OLS selection models. By explicitly incorporating the part-time/full-time wage differential into the structure of the model, we are not forcing the elasticity to capture large wage changes.

### Table 3.—Wage and Income Elasticities

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>OLS Selection Model</th>
<th>Tobit Model</th>
<th>IV Model</th>
<th>Dual-Error-Term Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross wage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PT wage</td>
<td>0.071</td>
<td>0.261</td>
<td>0.058</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.059)</td>
<td>(0.038)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>FT wage</td>
<td>0.091</td>
<td>0.338</td>
<td>0.076</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.076)</td>
<td>(0.049)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>Actual wage</td>
<td>0.073</td>
<td>0.271</td>
<td>0.061</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.061)</td>
<td>(0.039)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Compensated wage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PT wage</td>
<td>0.687</td>
<td>1.658</td>
<td>1.425</td>
<td>1.895</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.126)</td>
<td>(0.075)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>FT wage</td>
<td>0.890</td>
<td>2.148</td>
<td>1.846</td>
<td>2.454</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.163)</td>
<td>(0.098)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Actual wage</td>
<td>0.713</td>
<td>1.720</td>
<td>1.478</td>
<td>1.967</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.131)</td>
<td>(0.078)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Total income</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PT wage</td>
<td>−0.616</td>
<td>−1.397</td>
<td>−1.366</td>
<td>−1.840</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.108)</td>
<td>(0.108)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>FT wage</td>
<td>−0.798</td>
<td>−1.810</td>
<td>−1.770</td>
<td>−2.383</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.142)</td>
<td>(0.140)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>Actual wage</td>
<td>−0.639</td>
<td>−1.449</td>
<td>−1.417</td>
<td>−1.909</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.112)</td>
<td>(0.112)</td>
<td>(0.071)</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses (see Greene (1993, chap. 7)). Elasticities are calculated at sample means for workers (\(\bar{W}=11.09, \bar{W}_{P}=8.56, \bar{W}_{act}=8.88, H_{(intens)}=3.639\)). The following formula, derived from the Slutsky equation, is used to calculate the elasticities:

\[
\frac{\partial (\bar{W}/\bar{H})}{\partial (\bar{W}/\bar{H})} = \frac{(\bar{W}/\bar{H})(\bar{W}/\bar{H})}{\bar{W}} - \frac{1}{\bar{H}} \frac{\partial \bar{W}}{\partial \bar{H}} + \frac{\partial \bar{W}}{\partial \bar{H}}.
\]

### References

28 See Moffitt (1986, 1990) for a general discussion of the role of the two error terms in the estimation of labor supply with a nonlinear budget constraint. Note that unlike the typical continuous nonconvex budget constraint, we expect a fair amount of clumping at the kink point on the budget constraint that incorporates a part-time/full-time wage differential, which happens to be discontinuous at the kink (see section II).

29 Comparing the dual-error-term parameter estimates and the resulting labor supply elasticities to the IV results allows us to isolate the impact of accounting for a structural nonlinearity in the budget constraint. Both methods endogenize the segment choice; they differ primarily in terms of their structural assumptions. In addition, comparisons made with results obtained by others are useful only to the extent to which they illustrate that the results obtained here are not out of line with the rest of the literature. The elasticities presented here are well within the bounds established by the survey of women’s labor supply by Killingsworth and Heckman (1986).

30 Moffitt (1984b) also estimates wage elasticities that decrease in size as he allows for nonlinearities in the budget constraint. He estimates a 0.78 gross wage elasticity when assuming a linear budget constraint, and a 0.43 gross wage elasticity when the wage is specified as a quadratic function of hours worked.
changes that would occur as hours move from part-time to full-time. The small gross wage elasticity of the IV model indicates that endogenizing the wage goes a long way to accounting for this discrete jump in hours from part-time to full-time.

C. Evaluating Goodness of Fit

Theoretically the dual-error-term model is the preferred specification as it explicitly models the underlying budget constraint. However, it is natural to want some confirmation of the “goodness of fit” for this model and some means of comparing its performance to the IV and tobit models, which are computationally much simpler. For the OLS models the conventional $R^2$ measure provides an indication of the goodness of fit. However, for the dual-error-term and tobit models, the usual $R^2$ term is meaningless. Greene (1993) suggests that an analog to the $R^2$ in OLS is the likelihood ratio index ($LRI$) calculated as

$$LRI = 1 - \frac{\ln L}{\ln L_0}$$

(5)

where $\ln L$ is the log of the likelihood function evaluated at the parameter estimates and $\ln L_0$ is the log of the likelihood function evaluated with all the slope coefficients set to zero. This measure is also bounded by 0 and 1, though not directly comparable to the traditional $R^2$. For each of the models we calculated the $LRI$ for comparison purposes since the OLS and IV results can also be obtained via maximum likelihood assuming normally distributed random-error terms. Based on the $LRI$ values reported in table 4, the dual-error-term model does slightly better than the IV model and substantially better than the OLS model.

Another test of the strength of a model is how well it predicts within sample. The hours distribution predicted from the tobit and dual-error-term models are found in figure 3 along with the frequency distribution of actual hours worked. The predicted hours distributions for the maximum-likelihood models (the tobit and dual-error-term models) are obtained by plotting the implied hours distributions, which makes use of the structure of the likelihood function maximized for each model. The implied hours distribution is merely a plot of the probabilities of the mean person being observed working in each five-hour interval, calculated using the appropriate pieces of the likelihood function. (See Moffitt (1984b) for a more detailed description of calculating the implied hours distribution.) We do not include plots of the OLS and IV models because these models are not designed to fit the distribution of hours. OLS, for example, just fits the mean of the data $E(H)$ for a given set of values of the $X$ variables, thus judging the OLS models relative to the tobit and dual-error-term models is incorrect, since OLS is not a technique that attempts to fit the hours distribution.

As can be seen in figure 3, the dual-error-term model does a fairly good job of predicting the actual distribution of hours compared to the tobit model. It appears that the structure imposed by the dual-error-term model is necessary to account for the discrete clumping of the hours distribution, especially in the full-time range.

VI. Concluding Remarks

The purpose of this paper is to examine the role the well-documented part-time/full-time wage differential plays in determining labor supply by structurally incorporating that wage differential into the budget set that decision makers face. The incidence of a higher wage being paid to full-time (high-hours) workers and a lower wage being paid to part-time (low-hours) workers also provides a natural setting in which to explore the importance of tied wage-hours offers in the estimation of labor supply. In addition, the discontinuity in the budget constraint generated by the part-time/full-time wage differential introduces a degree of difficulty in the estimation procedure and raises some estimation issues previously not encountered in the nonlinear budget set literature.

The incorporation of the part-time/full-time wage differential introduces two essential problems into the estimation of labor supply: the wage is determined endogenously and the discontinuity of the budget constraint implies “clumping” of the data. Endogenizing the wage results in lower gross wage elasticities than are estimated by a standard OLS selection or tobit specification. The dual-error-term model is shown to best “fit” the data and to provide a respectable prediction of actual hours.

While this paper enters new territory in the estimation of nonlinear budget constraints by combining a discontinuity in the budget set with a dual-error-term labor supply model, and while it provides some interesting insight as to the implication of the effect of tied wage–hours offers on female wage elasticities, there is at least one area where the analysis can be expanded. Some work has been done on the joint estimation of wages and hours (Moffitt (1984b) and Tummers and Woittiez (1991)). The part-time/full-time wage differential makes joint estimation of wages and hours extremely complicated, but worth pursuing in further explorations of part-time labor supply.

REFERENCES


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31 This index is also often referred to as MacFadden’s $R^2$. (See Maddala (1992, p. 334) and McFadden (1974).)
32 We are grateful to an anonymous referee for illuminating this point.
FEMALE LABOR SUPPLY WITH DISCONTINUOUS NONCONVEX BUDGET CONSTRAINT


———, *Limited-Dependent and Qualitative Variables in Econometrics* (Cambridge: Cambridge University Press, 1983).


APPENDIX

**Likelihood Function for the Dual-Error-Term Model**

The function describing the desired hours choice for workers is

\[
\begin{align*}
  h^* &= 168, & \text{if } k(F, 168) > k(F, H^*) \\
  h^* &= g(W_F, Y) + \epsilon_* + \epsilon, & \text{if } k(F, 168) > k(F, H^*) > m(P, F) \\
  h^* &= g(W_F, Y) + \epsilon_* + \epsilon, & \text{if } m(P, F) > k(P, 0) \\
  h &= 168 + \epsilon, & \text{if } k(F, 168) > k(F, 168) > \epsilon_* > k(F, H^*) \text{ or } \epsilon > k(P, 0) \\
  h &= g(W_F, Y) + \epsilon_* + \epsilon, & \text{if } k(F, H^*) > \epsilon_* > m(P, F) \\
  h &= g(W_F, Y) + \epsilon_* + \epsilon, & \text{if } m(P, F) \geq \epsilon_* > k(P, 0).
\end{align*}
\]
Letting $\epsilon_s$ and $\epsilon_l$ be distributed normally and independently with means zero and variances $\sigma_s^2$ and $\sigma_l^2$, the probability density of a given hours observation for a worker is

$$P(h) = P[\epsilon_s = h - 168, \epsilon_l \geq k(F, 168)] + P[\epsilon_s + \epsilon_l = k(F, h), k(F, 168) \geq \epsilon_s \geq k(F, H^*)] + P[\epsilon_s = h - H^*, k(F, H^*) > \epsilon_s > m(P, F)] + P[\epsilon_s + \epsilon_l = k(F, h), m(P, F) \geq \epsilon_s \geq k(F, 0)] \quad (A.3)$$

Since $\epsilon_s$ and $\epsilon_l$ are independently distributed as normal, $\epsilon_s + \epsilon_l \sim N(0, \sigma^2_s + \sigma^2_l)$. The contribution to likelihood of a labor market participant is the probability of observing a particular value for $h$, or

$$P(h) = \frac{1}{\sigma_s} \left[ h - 168 \right] - \Phi \left( \frac{[k(F, 168)]}{\sigma_s} \right) + \frac{1}{\sigma_s} \Phi \left( \frac{[k(F, h)]}{\sigma_s} \right) \frac{[k(F, 168) - \nu^2k(F, h)]}{\sigma_s(1 - \nu^2)^{1/2}} - \Phi \left( \frac{[k(F, H^*) - \nu^2k(F, h)]}{\sigma_s(1 - \nu^2)^{1/2}} \right) + \frac{1}{\sigma_s} \left[ h - H^* \right] \quad (A.4)$$

where $\sigma = \sigma_s/\sigma_l$ and $k(\cdot)$ and $m(\cdot)$ are as defined above. The contribution to likelihood made by nonparticipants is

$$P(h = 0) = \Phi \left[ \frac{[k(F, 168)]}{\sigma_s} \right] - \Phi \left( \frac{[k(F, 168)]}{\sigma_h} \right) + \frac{\nu^2k(F, 0)}{\sigma_s(1 - \nu^2)^{1/2}} - \frac{\nu^2k(F, 0)}{\sigma_h} = \Phi \left( \frac{[k(F, H^*) - \nu^2k(F, 0)]}{\sigma_s(1 - \nu^2)^{1/2}} \right) + \Phi \left( \frac{[k(F, H^*) - \nu^2k(F, 0)]}{\sigma_h} \right) \quad (A.5)$$

Note that the heterogeneity-error term does not enter the probability of being at the point of discontinuity since the number of hours at that point is fixed at $H^*$. Also, 168 hours is chosen as the upper limit because it represents total hours available in a week.

Also see Triest (1990).