3C Dependent Prices in the Many Good World


"There is not much virtue in simplicity if a result that holds in a model of two countries, two commodities, and two factors does not generalize in any meaningful way to higher dimensions." John Chipman (1987, p. 922).

Empirical Studies

● Leontief (1953): discovered that U.S. import-competing industries used 30 percent more capital per worker than export industries. This finding has come to be known as the Leontief Paradox.

● In all subsequent empirical studies, the number of industries has been much greater than the number of factors.

● For instance, Leontief’s (1956) second test included 192 industries, but only two factors, K and L.

● Similarly, in Stern and Maskus (1981) and Trefler (1993), the number of industries was much greater than that of factors.
The $n \times 2$ model does not predict that exports of a capital-abundant country will be capital intensive.

Leontief’s approach was not valid, because he expected the prediction of a $2 \times 2$ model to be borne out in his two empirical studies that included more than two industries.

Two stylized facts

Fact 1. The number of outputs, $n$, is much greater than that of factors, $m$, used to produce the outputs.

Fact 2. Typically, a trading country produces $k$ goods, $m < k < n$, and the $k/m$ ratio is closer to $n/m$ than to unity.

The relationship between inputs and outputs

$$A Y = V,$$  \hspace{1cm} (1)

where $A = [a_{ij}]$ is an $m \times n$ matrix, $Y$ is an $n \times 1$ output vector, and $V$ an $m \times 1$ input vector. The trade vector is

$$X = Y - C,$$  \hspace{1cm} (2)

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In the $3 \times 2$ case, the output vector $Y$ has one degree of freedom. As Leamer (1984, 1987) observed, in general the output vector will have $(n - m)$ degrees of freedom.
Leontief (1953): 38 traded sectors \( \Rightarrow n/m = 19 \).

Leontief’s (1956) 2\textsuperscript{nd} test: 192 industries in the U.S. trade pattern in 1951, the \( n/m = 96 \).

Stern and Maskus (1981): the Ricardian goods, the HO goods, and the Product Cycle goods. For HO goods, \( n/m \) was about 40.

Trefler (1993): 79 sectors and 10 factors \( \Rightarrow n/m = 8 \).

Long-run Indeterminacy of the Output Vector

The system of equations in (1) has \( (n - m) \) degrees of freedom and for all practical purposes, a country's output vector is indeterminate.

Ethier (1984, p. 143) suggested that commodity prices are not drawn from an urn but are interconnected.

Leamer’s (1984 book) Approach

The problem then is to choose the output vector \( y \) to:

\[
\text{maximize}\quad I = Py \quad \text{subject to:} \quad Ay = V,
\]

where \( P \) and \( y \) are \( n \times 1 \) vectors of exogenous prices and outputs, and \( V \) is an \( m \times 1 \) vector of fixed factor endowments.
● The Lagrangian function associated with this problem is:
\[ \mathcal{L} = \mathbf{Py} + \mathbf{W} [\mathbf{V} - \mathbf{Ay}] , \]  

(3)

where \( \mathbf{W} \) is an \( m \times 1 \) vector of Lagrange multipliers, reflecting the shadow prices of the internationally immobile inputs.

● Leamer (1987) shows that given an arbitrary price vector \( \mathbf{P} \), optimal outputs are positive only for \( m \) industries and the outputs of the remaining sectors equal zero.

● Choi (2004): The contrapositive of Leamer’s result is that if more than \( m \) goods are actually produced, then output prices must be dependent on each other.

● In all empirical studies, the \( k/m \) ratio has been closer to \( n/m \) than to unity. For instance, in Leontief’s first test, 35 industries were net exporters and three were net importers.

● This suggests that output prices are interlinked, and commodity prices move together, at least among the goods that are actually produced.
If $p_1$ rises, which price will adjust?

Either $p_2$ has to rise or $p_3$ has to fall, or both must occur. It follows that when the price of an extreme industry rises or falls, the price of an intermediate industry cannot remain constant without causing a further change in the price of the other extreme good.

Figure 1 shows the case where the price of the middle good also rises.
Figure 2 illustrates the case when $p_1$ and $p_2$ are firm, and hence $p_3$ has to adjust to survive.

Figure 3 examines the case where the price of an intermediate industry rises, shifting the unit value isoquant downward to $y_2$. If $p_1$ is held constant, then $p_3$ must rise, resulting in a factor price ratio, $(w/r)^a$. 
In the \( n \times 2 \) (\( n > 2 \)) model, if the price of one good changes, at most the price of only one other good (numéraire) can be held constant, but all other prices may change.

In the \( m \times n \) (\( n > m \)) model, following an increase in the price of one good, at most \((m - 1)\) output prices can be held constant. These \( m \) output prices completely determine the \( m \) factor prices, and the remaining \((n - m)\) output prices must adjust accordingly.
The Effects of Interdependent Output Prices on HO

- Each industry makes zero profit $\Rightarrow$ Output vector is indeterminate. $\Rightarrow$ trade vector is indeterminate. That is, the trade pattern cannot be predicted from the factor endowment vector.

- FPE: Not affected. For instance, in the $n \times 2$ world, two output prices completely determine the two factor prices. But this does not eliminate the possibility of no FPE.

- Rybczynski Theorem: Not defined.
- Stolper-Samuelson Theorem: The Question is not defined.

Effects of No Factor Price Equalization

![Diagram](image)

Figure 6. Factor content of trade without FPE.
Figure 6 illustrates that the sum of factors exported by both countries need not be zero when factor prices are not equalized.

If FPE does not hold, any analysis of factor contents of trade is meaningless. Both counties will claim they exported/imported different amounts of factors.

If FPE does not hold, and a FIR occurs, they will claim they exported the same factor.