5F. Strategic Trade Policy:  
To Subsidize or Not To Subsidize

Based on Brander and Spencer (JIE 1985)

- Scenario: a home firm (Boeing) and a foreign firm (Airbus) compete in exporting to a third country market (Asia).

Payoffs

- Scenario A: Nonintervention

<table>
<thead>
<tr>
<th></th>
<th>Boeing Enter</th>
<th>Boeing Not enter</th>
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<tbody>
<tr>
<td>Airbus Enter</td>
<td>-5, -5</td>
<td>50, 0</td>
</tr>
<tr>
<td>Airbus Not enter</td>
<td>0, 50</td>
<td>0, 0</td>
</tr>
</tbody>
</table>
Scenario B: Subsidize Airbus by 6.

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</thead>
<tbody>
<tr>
<td>Boeing</td>
<td>-5, -5+6</td>
<td>50,0</td>
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<tr>
<td>Enter</td>
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</tr>
<tr>
<td>Not enter</td>
<td>0,50+6</td>
<td>0,0</td>
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(solution: Entry is Airbus’s dominant strategy. Given that Airbus enters, Boeing’s solution is not to enter. Accordingly, subsidy affects the outcome.

(Definitions of x and y are reverse in Strategic Trade policies.)

\[ p = p(x + y) \] : inverse demand function.

\[ s = \text{subsidy per unit of home firm’s exports.} \]

\[ c(x) = \text{production cost} \]

Domestic firm’s profit

\[ \pi(x, y, s) = xp(x + y) - c(x) + sx. \] (1)
First order condition (FOC):

\[ \pi_x(x, y, s) = p(x + y) + xp'(x + y) - c' + s = 0. \] \hspace{1cm} (2)

FOC yields the domestic firm’s best response function 
\[ x = b(y, s) \] in terms of the foreign firm’s output and subsidy. Check the second order condition (SOC)

SOC:

\[ \pi_{xx}(x, y, s) = 2p' + xp'' - c'' < 0. \]

● Foreign firm’s profit

\[ \pi^*(x, y) = yp(x + y) - c^*y. \] \hspace{1cm} (3)

The export subsidy does not affect the foreign firm’s profit directly, but indirectly through the domestic firm’s output.

FOC:

\[ \pi_y^* = p(x + y) + yp' - c'(y) = 0. \] \hspace{1cm} (4)

FOC yields the foreign firm’s best response function, 
\[ y = b^*(x). \]

SOC:

\[ \pi_y^* = 2p' + yp'' - c^*'' = 0. \]
More assumptions about the property of \( \pi(.) \):

\[
\pi_{xy} = p' + xp'' < 0, \quad \pi_{xy}^* = p' + yp'' < 0,
\]

Also, note that

\[
\pi_{xx} = 2 p' + xp'' - c'' < p' + xp'' = \pi_{xy},
\]

Similarly,

\[
\pi_{yy}^* < \pi_{xy}^*.
\]

Thus,

\[
D = \pi_{xx} \pi_{yy}^* - (\pi_{xy} \pi_{xy}^*) > 0. \tag{5}
\]

If each profit function is concave and condition (5) holds globally, then there is a unique solution \((x, y)\).
Effects of Subsidy

The two FOCs are displayed again below for convenience.

\[
\pi_x(x, y, s) = p(x + y) + xp'(x + y) - c' + s = 0. \tag{2}
\]

\[
\pi_y^* = p(x + y) + yp' - c^*(y) = 0. \tag{4}
\]

To investigate the effect of an export subsidy, differentiate the following system with respect to \(s\):

\[
\begin{bmatrix}
\pi_{xx} & \pi_{xy} \\
\pi_{yx} & \pi_{yy}
\end{bmatrix}
\begin{bmatrix}
dx \\
\frac{dy}{ds}
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
0
\end{bmatrix}. \tag{6}
\]

Using Cramer’s rule, we have

\[
\frac{dx}{ds} = \frac{\begin{vmatrix}
-1 & \pi_{xy} \\
0 & \pi_{yy}^*
\end{vmatrix}}{D} = \frac{-\pi_{yy}^*}{D} > 0,
\]

\[
\frac{dy}{ds} = \frac{\begin{vmatrix}
\pi_{xx} & -1 \\
\pi_{xy}^* & 0
\end{vmatrix}}{D} = \frac{\pi_{xy}^*}{D} < 0, \tag{7}
\]

Thus, the change in total supply is
\[
\frac{d(x + y)}{ds} = \frac{-\pi_{yx}^* + \pi_{xy}^*}{D} > 0.
\]

It follows that the subsidy lowers the price.

Differentiating profits with respect to \( s \) gives

\[
\frac{d\pi}{ds} = \pi_s x_s + \pi_y y_s + \frac{\partial \pi}{\partial s} = 0 + xp'y_s + x > 0.
\]

\[
\frac{d\pi^*}{ds} = \pi_s^* x_s + \pi_y^* y_s = yp' < 0.
\]

**Proposition 1** (Brander and Spencer, 1985): Consider a scenario in which a domestic and a foreign firm compete in a third market. Then a subsidy decreases the price, increases domestic firm’s profit while lowering the foreign firm’s profit, i.e.,

\[
\frac{dp}{ds} < 0, \quad \frac{d\pi}{ds} > 0, \quad \frac{d\pi^*}{ds} < 0.
\]

Profit Shifting: The subsidy shifts profits from the foreign firm to the domestic firm.
Welfare

\[ G(s) = \pi(x, y, s) - sx. \]  \hfill (9)

Differentiating (9) with respect to \( s \) gives

\[ \frac{dG(s)}{ds} = xp'y_s - sx_s, \]  \hfill (10)

which is generally indeterminate. However, when evaluated at \( s = 0 \), it is positive.

That is, a subsidy first increases the surplus. But beyond a certain point, any additional subsidy may decrease subsidy. In other words, there is an optimal subsidy.

Remarks:

- Welfare should include consumer surplus or utility. Subsidies are paid also by consumers. If a single consumer is used for the third country, does it still pay to subsidize domestic producers?

- This model assumes a homogenous good. When differentiated goods are used, there will be two inverse demand functions, \( p(x, y, s) \) and \( q(x, y) \).

- Include domestic consumer welfare for US, Europe, and Asia. Use utility functions.
References

