Economics 571  
Problem Set #1  

(1) Suppose that the random variable $x$ has a triangular density over the interval $[-1, 1]$:

$$p(x) = [1 - |x|] \mathbb{I}(-1 \leq x \leq 1),$$

where $\mathbb{I}(\cdot)$ is an indicator function taking on the value of 1 if the event is true and otherwise is zero. (This indicator function simply serves to denote that the density is zero outside of the interval $[-1, 1]$).

(1a) Verify that this is a proper density function - that is, it integrates to one.  

(1b) Calculate $E(x)$.  

(1c) Calculate $E(x^2)$ and $\text{Var}(x)$.  

(1d) Calculate $\Pr(x \leq .75)$.  

(1d) Calculate $\Pr(-.75 \leq x \leq .75)$.  

*Note:* For these calculations is may prove to be easier to divide the integral into two pieces: one from $-1$ to 0, and the other from 0 to 1 in order to calculate the desired quantities.  

(2) Suppose that a pair or random variables $x$ and $y$ have the curved roof distribution, given as

$$f(x, y) = 3(x^2 + y)/11, \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 1.$$  

(a) Show that the marginal density of $x$, $p(x)$, is given as

$$p(x) = 3(2x^2 + 1)/22, \quad 0 \leq x \leq 2.$$  

(b) Derive the marginal density of $y$, $f(y)$.  

(c) Derive the conditional density $g(y|x)$ for $0 \leq x \leq 2$.  