Economics 571  
Problem Set #2

(1) The random variable $X$ is distributed as $X \sim N(3,16)$, a normal distribution with mean 3 and variance 16.

Calculate the following:

(a) $\Pr(X \leq 7)$.

(b) $\Pr(X > 5)$.

(c) $\Pr(|X| \leq 3)$.

(2) The random variable $X$ is distributed as $X \sim N(3,16)$. Let $Y = 3 - (1/4)X$. (Note: Linear functions of normal random variables are also normal. Thus, $Y$ will also have a normal distribution). Calculate the following:

(a) $\Pr(3.25 < Y < 4.25)$.

(b) $\Pr(Y > X)$. 

(3) Suppose you are charged with designing a survey to recover a parameter of interest $\theta$. You can ask $n$ possible people for information regarding the value of $\theta$. You also know that there are two different groups of people comprising the population. Let $x_1$ denote a response from group 1 and $x_2$ denote a response from group 2.

Group 1 is known to give a biased report of $\theta$, and specifically, the responses from group 1 are known to be generated by:

$$x_{1i} \sim N(\theta + c, \sigma_1^2), \quad i = 1, 2, \ldots, n_1,$$

with $n_1$ denoting the number of people sampled from group 1 and $c$ some constant.

Group 2 is known to give an unbiased report of $\theta$, and the responses from group 2 are known to be generated by:

$$x_{2i} \sim N(\theta, \sigma_2^2), \quad i = n_1 + 1, n_1 + 2, \ldots, n.$$

However, $\sigma_2^2 = \sigma_1^2 + d$, for some constant $d > 0$ so that the responses from group 2 are more variable than those from group 1.

You use the estimator (where all $x_i$ are assumed independent of one another):

$$\hat{\theta} = \frac{\sum_{i=1}^{n_1} x_{1i} + \sum_{i=n_1+1}^{n} x_{2i}}{n}.$$

Using mean squared error (MSE) as a criterion function for determining the optimal $\hat{\theta}$, find the $n_1$ value that minimizes MSE and comment on the result you obtain. You do NOT need to consider feasibility constraints in your solution, i.e., that $n_1$ is integer-valued or that $0 < n_1 < n$.

(4) Prove or provide a counterexample to the following statement: A consistent estimator of a parameter $\theta$ is also an unbiased estimator of $\theta$. 
