

Economics 571  
Problem Set #1

(1) Suppose that a random variable  $x$  has an *exponential* density with parameter  $\lambda > 0$ :

$$p(x) = \lambda \exp(-\lambda x), \quad x \geq 0.$$

Thus, the distribution places no mass to the left of zero.

Show that the exponential density is a proper density function. (Note: the upper limit of integration here will be equal to  $\infty$ .)

(2) Suppose that  $x$  has a *rectangular* or *uniform* distribution on the interval  $[0, 2]$ :

$$p(x) = 1/2 \quad 0 \leq x \leq 2.$$

(2a) Calculate  $E(x)$ .

(2b) Calculate  $E(x^2)$  and  $\text{Var}(x)$ .

(2c) Calculate  $\Pr(x \leq 1.5)$ .

(3) Consider the *Pareto* density:

$$f(x) = \frac{\beta \alpha^\beta}{x^{\beta+1}}, \quad 0 < \alpha < x < \infty, \quad \beta > 0.$$

(Make sure to note the support of  $x$  as a function of  $\alpha$ .)

(3a) Verify that this is a proper density function.

(3b) Derive the cdf,  $F(c)$ , for  $c > \alpha$ .

(3c) Derive the mean and variance of  $x$ , and clearly state what, if any, conditions must be satisfied for these to exist.