

# Sample Midterm Solution

|                              | Estimator | Constant | Stochastic variable | Parameter |
|------------------------------|-----------|----------|---------------------|-----------|
| 5                            |           | X        |                     |           |
| $k$                          | X         |          | X                   |           |
| $e$                          |           |          | X                   |           |
| $\rho$                       |           |          | X                   |           |
| $a^2$                        |           | X        |                     | X         |
| SSR                          |           |          | X                   |           |
| $K$                          |           | X        |                     | X         |
| $s^2 \hat{Y}X'X\hat{P}^{-1}$ | X         |          | X                   |           |

1. (35 points) Consider the least squares residual vector  $e$  from the regression of  $y$  on  $X$ , where the errors  $P$  have variance  $VPP = a^2I$ . Assume that  $X$  is *not* stochastic. Show that the variance of any element of  $e$ , say  $e_j$ , is less than or equal to  $a^2$ . Hint:  $\hat{a}P_X \hat{a}_{jj} = X^j \hat{Y}X'X\hat{P}^{-1} \hat{Y}X^j$ , where  $X^j$  is the  $j$ 'th ROW of  $X$ . Show all steps.

$$e = M_X y = M_X Y - X\beta = M_X P$$

$$V e = M_X V P P M_X = a^2 M_X = a^2 I - a^2 P_X$$

$$V e_j = a^2 - a^2 X^j \hat{Y}X'X\hat{P}^{-1} \hat{Y}X^j$$

Since  $\hat{Y}X'X\hat{P}^{-1}$  is p.d. so is  $\hat{Y}X'X\hat{P}^{-1}$  and therefore  $X^j \hat{Y}X'X\hat{P}^{-1} \hat{Y}X^j \geq 0$  and hence  $V e_j \leq a^2$ .

3. (30 points) Data on working men was used to estimate the following equation:

$$educ = 10.36 + 0.094 \text{ sibs} + 0.131 \text{ meduc} + 0.210 \text{ feduc}$$

$$n = 733, R^2 = 0.214$$

where  $educ$  is years of schooling,  $sibs$  is number of siblings,  $meduc$  is mother's years of schooling and  $feduc$  is father's years of schooling.

- Discuss the interpretation of the coefficient on  $sibs$ . For each sibling you have, on average you will have 1/10 of a year less of schooling. It seems that the more children, the less education for each.
- Holding  $meduc$  and  $feduc$  fixed, how much does  $sibs$  have to increase to reduce the predicted years of schooling by one year? It has to increase by  $\frac{1}{0.094} = 10.6$ , so the effect is not that severe!
- Suppose Harvey has no siblings and his parents each have 12 years of schooling. John has no siblings and his parents each have 16 years of schooling. What is the predicted difference in years of schooling between Harvey and John? Harvey:  $educ = 10.36 + 0.094 \cdot 0 + 0.131 \cdot 12 + 0.210 \cdot 12 = 14.452$  John:

$educ = 10.36 + 0.094 \cdot 60 + 0.131 \cdot 16 + 0.210 \cdot 16 = 15.816$ . So there's approximately 1.5 years of difference between their education levels.