Final
Econ 573
Professor Bunzel
Thursday, May 3rd, 2001

You have 2 hours to write this exam. The exam consists of 3 questions. The maximum possible score is 100 points. There are 2 pages to this exam including this one. If there is any question you do not understand please ask me for clarification. Also remember that partial credit is given.

PLEASE remember to put your name and social security number on the answers!!

GOOD LUCK!!!!
1. Assume
\[ y_t = KX_t + u_t, \quad X_t = t, \quad t = 1, \ldots, T \quad u_t \sim iid \sim \mathcal{N}(0, \sigma^2) \]
Note that this is just regressing \( y \) on a time trend.
Let \( \hat{K} \) be the OLS estimator of \( K \).

a. Show that the asymptotic distribution of \( \sqrt{T} (\hat{K} - K) \) is degenerate (i.e. the variance of \( \sqrt{T} \hat{K} \) converges to 0).

b. Find the asymptotic distribution of \( T^{1/2} (\hat{K} - K) \).

Hint:
\[
\frac{1}{T} \sum_{t=1}^{T} t^2 = \frac{\sigma^2 T + 1}{6} + \frac{2T + 1}{6}
\]

2. In a study of housing expenditures the data has been divided into \( G \) income intervals each with \( g_i, i = 1, \ldots, G \) observations. Assume that the following model is valid for each income interval
\[ y_{it} = X_{it}K + u_{it}, \quad u_{it} \sim N(0, \sigma_i^2) \]

a. Derive an estimator for \( K \).

b. Determine the properties of your estimator

c. Derive the LR test for the hypothesis
\[
H_0 : \quad a_1^2 = a_2^2 = \ldots = a_G^2 = a^2 \\
H_1 : \quad a_i^2 \neq a^2 \quad \text{for at least one } i
\]
3. Suppose that, for a given state in the United States, you wish to use annual time series data to estimate the effect of the state-level minimum wage on the employment of those 18 to 25 years old ($EMP$). A simple model is

$$g_{EMP_t} = K_0 + K_1 g_{MIN_t} + K_2 g_{POP_t} + K_3 g_{GSP_t} + K_4 g_{GDP_t} + u_t,$$

where $MIN_t$ is the minimum wage, in real dollars, $POP_t$ is the population from 18 to 25 years old, $GSP_t$ is gross state product, and $GDP_t$ is U.S. gross domestic product. The $g$ prefix indicates the growth rate from year $t-1$ to year $t$, which would typically be approximated by the difference in the logs.

a. If we are worried that the state chooses its minimum wage partly based on unobserved (to us) factors that affect youth employment, what is the problem with OLS estimation?

b. Let $USMIN_t$ be the U.S. minimum wage, which is also measured in real terms. Do you think $gUSMIN_t$ is uncorrelated with it?

c. By law, any states minimum wage must be at least as large as the U.S. minimum. Explain why this makes $gUSMIN_t$ a potential IV candidate for $gMIN_t$. 