Problemset 8

1. This problem walks you through the asymptotic distribution of the OLS estimator when the regressors are stochastic. Consider the following model with *stochastic* regressors:

\[ y = X\beta + \varepsilon \]

Maintain the following assumptions:

- The sequence \( \{(\varepsilon_i, x_i)\} \) is iid
- \( E(\varepsilon_i | x_i) = 0 \)
- \( V(\varepsilon | X) = \sigma^2 I \)
- \( E(x_i' x_i) = Q \)
- \( \plim \left( \frac{1}{n} X' X \right) = Q \)

(a) Using the law of iterated expectations, show that

\[ E(x_i \varepsilon_i) = 0 \]

where \( x_i : k \times 1 \) is the vector of regressors for observation \( i \).

(b) Find the variance of \( x_i \varepsilon_i \).

(c) What is the variance-covariance matrix of \( X' \varepsilon \)?

(d) Use the Lindenberg-Levy Central Limit Theorem to find the asymptotic distribution of \( \sum x_i \varepsilon_i \). (This will need to be correctly scaled)

(e) Finally, find the asymptotic distribution of \( \hat{\beta} \).

2. In the dataset PS8dat.txt, you will find data on consumption and income. With it carry out the following procedures:

(a) Estimate the standard model of consumption.

(b) To take into account possible correlation between \( Y \) and the errors, estimate the model using IV estimation. Do this with the two instruments: \( Y \) last period, \( Y \) two periods earlier. (\( Y \) lagged once and twice)

(c) Considering the estimates using the two methods and their standard errors, do you think that the estimates from the two different methods are very different?

(d) Carry out the Hausman test to see whether there actually is correlation between \( Y \) and the errors.