Econ 573  
Problem Set #8  
Due: April 7, 1999

For each of the following problems, please provide detailed written responses and explain all derivations. Also, you should attach copies of computer programs and output if required to answer a question.

1) Suppose that you have developed a linear regression model of the form

\[ y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \epsilon \]  

(1)

where

\[ \text{Var}(\epsilon | X) = \sigma^2 x_2^2 \]  

(2)

a) How would you correct heteroskedasticity in this case? In particular, what transformation of the model would yield a specification consistent with the classical linear regression model from chapter 6 of Greene?

b) If you estimated (1) using GLS and the true error specification is \( \text{Var}(\epsilon | X) = \sigma^2 x_1^2 \), would your estimates of \( \beta \) and \( \sigma^2 \) be

i) Unbiased?

ii) Efficient?

iii) Consistent?

Carefully justify your claims.

2) As noted in class, FGLS need not be preferred in small sample applications. Demonstrate this result by conducting a small-scale Monte Carlo investigation (using TSP or GAUSS) into the small sample properties of OLS, GLS, and FGLS. Specifically, suppose that you have the linear regression model:

\[ y_i = \alpha + \beta x_i + \epsilon_i \]  

(3)

where

\[ \epsilon_i \sim N(0, \sigma^2 \exp(\gamma z_i)) \]  

(4)

Let \( \alpha = 0, \beta = 1, \) and \( \sigma = 1. \) Assume that \( x_i \) and \( z_i \) are independent and uniformly distributed in the population.

a) Start by assuming that \( \gamma = 0.05. \)
i) Generate a sample of \( N=20 \) observations drawn from this model. You will have to first draw values for \( x_i \) and \( z_i \) and then construct \( \varepsilon_i \) using the distributional assumption (4). Finally, generate values for \( y_i \) using equation (3).

ii) For this sample, estimate the parameters of the model using OLS, FGLS, and GLS.

iii) Repeat the process in (i) and (ii) a total of \( M=100 \) times, storing your estimates of the parameters of the model (i.e., \( \alpha, \beta, \) and \( \sigma \) ). Compare the mean square error of these estimates among the three estimators. In particular, how do OLS and FGLS compare? How much is gained by knowing the true variance structure (i.e., compare OLS and GLS)?

b) Repeat the above exercise using

i) \( N=100 \) and \( \gamma=.05 \).

ii) \( N=20 \) and \( \gamma=1 \).

iii) \( N=100 \) and \( \gamma=1 \).

How do your findings change?

3) Suppose that \( y_t = \alpha + \beta z_t + \varepsilon_t \), and \( \varepsilon_t = \rho \varepsilon_{t-1} + u_t \), with \( E u_t = 0, \text{Var } u_t = \sigma^2 \), and \( Cov(u_t, u_j) = 0 \) if \( i \neq j \). Given that \( \rho = 0.6 \)

\[
\begin{align*}
y &= \begin{bmatrix} 2 \\ 5 \\ 2 \\ 2 \\ 1 \\ 10 \\ 7 \end{bmatrix} \\
X &= \begin{bmatrix} i & z \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 1 \\ 1 & 1 \\ 10 & 10 \\ 7 & 7 \end{bmatrix}
\end{align*}
\]

(5)

a) Find the transformed observations \( y^* \) and \( X^* \).

b) Find the GLS estimates of \( \alpha \) and \( \beta \).

c) Find the Durbin Watson statistic (assuming \( \rho = 0 \) ) and test for first order autocorrelations.

Do not use TSP or GAUSS regression routines to carry out the above exercises, though you can use them to check your results.

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1 When conducting FGLS, assume that you know the form of the heteroskedasticity, but not the value of \( \gamma \), whereas when you conduct GLS, assume that you know \( \gamma \).
4) Greene, page 610, problems 5 and 6.