Uncertainty in the MC function

- Uncertainty in MC function affects quantity and price instruments differently.

\[ \text{WL}_{\text{tax}} = ABE - ACE \]  
\[ = \frac{1}{2} \Delta t_t \Delta Q_t - ACE \]  

where \( \Delta Q_t \equiv Q^E - Q^t \) and \( \Delta t_t \equiv t^t - t^E \)

- Let

\[ \eta_d = -\frac{\Delta Q_t}{\Delta t} \frac{t^E}{Q^E} \Rightarrow \Delta t_t = -\frac{\Delta Q_t t^E}{Q^E \eta_d} \]  

- Then

\[ \text{WL}_{\text{tax}} = -\frac{1}{2} \frac{(\Delta Q_t)^2 t^E}{Q^E \eta_d} - ACE \]
The Size of the Welfare Loss (cont’d)

- The size of welfare loss from the incorrect permit (or quantity) signal is given by:

\[
WL_q = ADE - ACE 
\]

\[
= \frac{1}{2} \Delta t_q \Delta Q_q - ACE
\]  

where \( \Delta Q_q \equiv Q^E - Q^t \) and \( \Delta t_q \equiv (D - E) \)

- Let

\[
\eta_s = \frac{\Delta Q_q}{\Delta t_q} \frac{t^E}{Q^E} \Rightarrow \Delta t_q = \frac{-\Delta Q_q t^E}{Q^E \eta_s}
\]

- Then

\[
WL_q = \frac{1}{2} \left( \frac{(\Delta Q_s)^2 t^E}{Q^E \eta_s} - ACE \right)
\]

The Relative Impact

- Combining these results, we have

\[
WL_{tax} - WL_q = -\frac{1}{2} \left( \frac{(\Delta Q_t)^2 t^E}{Q^E \eta_d} - \frac{1}{2} \frac{(\Delta Q_s)^2 t^E}{Q^E \eta_s} \right)
\]

\[
= \frac{1}{2} \frac{(\Delta Q_s)^2 t^E}{Q^E} \left( \frac{1}{\eta_d} + \frac{1}{\eta_s} \right)
\]

- The implication here is

1. If \( |\eta_d| = \eta_s \), then the welfare loss is identical under price and quantity instruments.
2. If \( |\eta_d| < \eta_s \) quantity instruments will be preferable.

B&O Prop. 3: ceteris paribus, the steeper the slope of the MB function, the smaller will be the distortion resulting from a regulatory error in the cost function from a permit system and the greater the distortion from a fee system.
3. If \( |\eta_d| > \eta_s \) price instruments will be preferable.

B&O Prop. 4: ceteris paribus, the reverse of Proposition 3 is true.