Econ 600
Schroeter
Fall 2004

Exam #1

Do all three problems. Weights: #1 - 25%, #2 - 35%, #3 - 40%. Closed book, closed notes. Be sure your answers are presented in a neat and well-organized manner.

1. Answer **True** or **False** for each of the following statements. If the statement is false, indicate how it could be changed to a true statement with a small change in the wording.

a. Given $F:\Re^n \to \Re$ differentiable. $F(\cdot)$ is concave if and only if

$$F(u) < F(v) + \frac{\partial F}{\partial x}(v)(u-v) \quad for\ all\ u,v \in \Re^n, u \neq v.$$

b. For $F:\Re \to \Re$, if $F(\cdot)$ is either strictly increasing or strictly decreasing, then $F(\cdot)$ is strictly quasi-concave.

c. $F:\Re^n \to \Re$ is differentiable and $x^* \in \Re^n$. If $\frac{\partial F}{\partial x}(x^*) = 0$ and $\frac{\partial^2 F}{\partial x^2}(x^*)$ is negative definite then x^* is a strict local maximum of $F(\cdot)$.

d. In an equality-constrained optimization problem with 3 choice variables (n = 3) and 2 constraints (m = 2), the bordered Hessian is a 5 x 5 matrix and the second order necessary condition is a single sign restriction on the determinant of the entire bordered Hessian.

e. Let $F:\Re^n \to \Re$ and $g:\Re^n \to \Re$ be differentiable and let $x^* \in \Re^n$ be a point such that $\frac{\partial g}{\partial x}(x^*) \neq 0$. Then x^* is a local solution to

$$\max_{w.r.t.\ x} F(x) \quad such\ that \quad g(x) = b$$

if and only if there exists $\lambda^* \in \Re$ such that (x^*, λ^*) is a stationary point of

$$L(x,\lambda) \equiv F(x) + \lambda[b - g(x)].$$

2. Consider the problem:

$$\min_{w.r.t.\, x} a \cdot x \quad such\ that \quad x \in U,$$

where x is an n x 1 vector of choice variables, a is a 1 x n vector of parameters, and $U \subset \Re^n$. Suppose that the problem has a global solution, $x^*(a)$, for all $a \in \Re^n$, and define the value function $F^*(a) \equiv a \cdot x^*(a)$.

(Some facts that you don't need to prove: A sufficient condition for the problem to have a global solution for all $a \in \Re^n$ is that U is closed and bounded. While the optimal value of x, $x^*(a)$, need not be unique for a given a, the value function is uniquely defined. That is, if, for a given a, there are multiple optimal x^*s, they all yield the same value of the objective function.)

Prove that $F^* : \Re^n \to \Re$ is concave.

3. Consider the following equality-constrained maximization problem:

$$\max_{w.r.t.\, x_1, x_2, x_3} f(x_1, x_2, x_3) \quad such\ that \quad g(x_1, x_2, x_3) = b \qquad (i.)$$

where $f : \Re^3 \to \Re$ and $g : \Re^3 \to \Re$ are differentiable functions and b is a constant. Also consider the unconstrained optimization problem:

$$\max_{w.r.t.\, x_1, x_2} F(x_1, x_2) \qquad (ii.)$$

where $F : \Re^2 \to \Re$ is differentiable.

a. Write down the Lagrangian and the first-order conditions for problem (*i.*).

b. Write down the first-order conditions for problem (*ii.*).

Now suppose that $g(x_1, x_2, x_3) = x_3 - h(x_1, x_2)$ where $h : \Re^2 \to \Re$ is differentiable, and that $F(x_1, x_2) \equiv f(x_1, x_2, h(x_1, x_2) + b)$. For this case, show that

c. if (x_1^*, x_2^*, x_3^*) satisfies the first-order conditions for problem (*i.*), then (x_1^*, x_2^*) satisfies the first-order conditions for problem (*ii.*).

d. if (x_1^*, x_2^*) satisfies the first-order conditions for problem (*ii.*), then $(x_1^*, x_2^*, h(x_1^*, x_2^*) + b)$ satisfies the first-order conditions for problem (*i*).