Econ 600
Schroeter
Fall 2011

Exam #2

Do all four problems. Weights: #1 – 30%, #2 – 20%, #3 – 20%, #4 – 30%. Closed book, open notes. Be sure your answers are presented in a neat and well-organized manner.

1. Answer **True** or **False** for each of the following statements. If the statement is false, indicate how it could be changed to a true statement with a small change in wording.

a. Let $F:\Re^n \to \Re$ be differentiable and quasi-concave. Let a be an *n*-dimensional (row) vector with $a \neq 0$. Suppose there exist $\lambda^* \in \Re$ and $x^* \in \Re^n$ such that $a \cdot x^* = 0$ and

$$\frac{\partial F}{\partial x}\left(x^*\right) = \lambda^* a .$$

Then x^* is a global solution to $\max_{w.r.t.\ x} F(x)$ such that $a \cdot x = 0$.

b. Let $F:\Re^n \to \Re$ be differentiable and quasi-concave. Suppose there exists $x^* \in \Re^n$ such that

$$\frac{\partial F}{\partial x}\left(x^*\right) = 0 .$$

Then x^* is a global solution to $\max_{w.r.t.\ x} F(x)$.

c. Let $F:\Re^n \to \Re$ be differentiable and let $x^* \in \Re^n$. If $\frac{\partial^2 F}{\partial x^2}\left(x^*\right)$ is negative definite then $\det \frac{\partial^2 F}{\partial x^2}\left(x^*\right) < 0$.

d. $F:\Re \text{ x } \Re \to \Re$ is differentiable. For each value of the scalar parameter *a*, $\max_{w.r.t.\ x} F(x; a)$ has a regular solution at $x^*(a)$. Define: $F^*(a) \equiv F\left(x^*(a); a\right)$. Then, for each value of the parameter, $a = a_0$:

$$\frac{\partial}{\partial a} F\left(x^*(a_0); a_0\right) = \frac{d}{da} F^*(a_0).$$

e. Every bounded monotone sequence in $\Re^1$ is a Cauchy sequence.

2. Let B be a compact (closed and bounded) subset of $\Re^p$. Let $f:\Re^p \to \Re^q$ be continuous. Define:

$$C = f(B) \equiv \left\{ y \in \Re^q : y = f(x) \text{ for some } x \in B \right\}.$$

Prove that C is closed. (It's also bounded, and therefore compact, but you don't have to prove this.)

Hint: Use the Bolzano-Weierstrass Theorem.

3. Let $F:\Re^n \to \Re$ be concave and let $g:\Re^n \to \Re$ be convex. Assume that the problem:

$$\max_{w.r.t.\ x} F(x) \text{ subject to } g(x) \le c\,,$$

has a global solution, denoted $x^*(c)$, for every $c \in \Re$. Define the value function $F^*:\Re \to \Re$ by $F^*(c) \equiv F\left(x^*(c)\right)$.

Prove that $F^*(\cdot)$ is concave.

4. Consider the problem:

$$\begin{aligned} \max_{w.r.t.\ x_1,\, x_2,\, x_3} \ln x_1 + \ln x_2 + \ln x_3 \quad \text{subject to} \quad & x_1, x_2, x_3 \ge 0 \\ & ax_1 + bx_2 + cx_3 \le R \\ \text{and} \quad & x_1 + x_2 \le S \end{aligned}$$

where $a, b, c, R,$ and S are positive parameters.

a.) Write down the Lagrangian and the Kuhn-Tucker conditions for this problem.

b.) Find a restriction on the parameter values that will insure that the second inequality constraint ($x_1 + x_2 \le S$) will not be binding at the optimum. Solve explicitly for optimal values of the choice variables in this case.