Econ 600
Schroeter
Fall 2006

Exam #2

Do all four problems. They will be equally-weighted. Closed book, open notes.
Be sure your answers are presented in a neat and well-organized manner.

1. Answer **True** or **False** for each of the following statements. If the statement is false, indicate how it could be changed to a true statement with a small change in the text.

a. If $F:\Re^n \to \Re$ is strictly concave then it is strictly quasi-concave.

b. Given $F:\Re^n \to \Re$ differentiable and $x^* \in \Re^n$ such that

$$\frac{\partial F}{\partial x}\left(x^*\right)=0 \quad and \quad \frac{\partial^2 F}{\partial x^2}\left(x^*\right) \text{ is negative definite.}$$

Then x^* is a strict local maximum of $F(\cdot)$.

c. Given $F:\Re^n \to \Re$ differentiable, let x^* be a local solution to

$$\min_{w.r.t.\ x} F(x) \quad subject\ to \quad x \in \Re^n_{+.}$$

Then, for $i=1,2,\ldots,n$: $\quad x_i^* \frac{\partial F}{\partial x_i}\left(x^*\right)=0.$

d. Let $F:\Re^n \to \Re$ and $g:\Re^n \to \Re$ be differentiable and let $b \in \Re$. Consider the problem:

$$\max_{w.r.t.\ x} F(x) \quad subject\ to \quad g(x) \le b \quad and \quad x_1, x_2, \ldots, x_n \ge 0.$$

If $x^* \in \Re^n$ is a local solution at which the non-degenerate constraint qualification is satisfied then, there exists $\lambda^* \ge 0$ such that, for each $i=1,2,\ldots,n$:

$$\frac{\partial F}{\partial x_i}\left(x^*\right)-\lambda^* \frac{\partial g}{\partial x_i}\left(x^*\right)=0.$$

e. A sequence in $\Re^n$ converges if and only if it is Cauchy.

2. Consider the following consumer utility maximization problem:

$$\max_{w.r.t.\ x_1,\, x_2} x_2 e^{x_1} \quad \textit{subject to } p_1 x_1 + p_2 x_2 \leq M; \quad x_1 \geq 0,\ x_2 \geq 0,$$

where x_1 and x_2 are quantities of goods 1 and 2, and p_1, p_2, and M are strictly positive prices and income.

a. Write down the Kuhn-Tucker conditions for this problem.

b. A solution "type" is characterized by the pattern of equalities in the inequality and non-negativity constraints at the optimum. (For example, solutions in which the budget constraint is binding and each of the non-negativity constraints is slack constitute one "type.") Use the Kuhn-Tucker conditions to show which solution types are possible and which are not possible. For each solution type that is possible:

i. Identify the subset of price-income space in which it occurs.

ii. Solve for the demand functions.

3. Given $A \subset \Re^n$, we say that $x \in A$ is a *cluster point of A* if, for all $\varepsilon > 0$, $B_\varepsilon(x)$ contains at least one point of A other than x.

Let *B* be an infinite compact set in $\Re^n$. Use the Bolzano-Weierstrass theorem to prove that *B* has a cluster point.

4. A firm produces output y using inputs x_1 and x_2 via the production function $y = f(x_1, x_2)$. The prices of both inputs are normalized at 1 and *p* denotes the price of output. Define the profit function: $\pi(p; y, x_1, x_2) \equiv py - x_1 - x_2$, and consider the following long- and short-run profit maximization problems:

Long-run problem: $\max\limits_{w.r.t.\ y, x_1, x_2;\ given\ p} \pi(p; y, x_1, x_2) \quad \textit{subject to} \quad y = f(x_1, x_2).$

Solutions: $y^*(p), x_1^*(p), x_2^*(p).$ Value function: $\pi^*(p).$

Short-run problem: $\max\limits_{w.r.t.\ y, x_1;\ given\ p, x_2} \pi(p; y, x_1, x_2) \quad \textit{subject to} \quad y = f(x_1, x_2).$

Solutions: $y^s(p, x_2), x_1^s(p, x_2).$ Value function: $\pi^s(p, x_2).$

a. Letting p^0 denote a specific value for *p*, sketch a graph showing the general relationship among the following functions of *p*:

$$\pi\left(p; y^*(p^0), x_1^*(p^0), x_2^*(p^0)\right), \quad \pi^s\left(p; x_2^*(p^0)\right), \quad \pi^*(p).$$

Explain the logic behind the relationship evident in the graph.

b. Based on the appearance of the graph, establish a relationship between the slopes of the long-run and short-run supply curves.