Econ 600
Schroeter
Fall 2007

Homework

1. Given differentiable functions:

$$F(x;\alpha): \Re^n x \Re^m \to \Re, \quad g(x;\alpha): \Re^n x \Re^m \to \Re, \quad and \quad h(x;\alpha): \Re^n x \Re^m \to \Re,$$

consider the following optimization problems:

$$(i) \qquad \max_{w.r.t.\ x,\ given\ \alpha} F(x;\alpha) \quad subject\ to \quad g(x;\alpha) = 0$$

$$(ii) \qquad \max_{w.r.t.\ x,\ given\ \alpha} F(x;\alpha) \quad subject\ to \quad g(x;\alpha) = 0 \quad and \quad h(x;\alpha) = 0$$

In these problems, $x \in \Re^n$ (where $n > 2$) is a vector of choice variables and $\alpha \in A$, an open subset of $\Re^m$, is a vector of parameters. Assume that each problem has a strict global solution for each $\alpha \in A$ and denote the optimal values of the choice variables, the equilibrium values for the problems' Lagrange multipliers, and the value functions (all assumed differentiable functions of α) as follows:

$$(i) \qquad x^*(\alpha), \quad \lambda^*(\alpha), \quad F^*(\alpha) \equiv F(x^*(\alpha);\alpha)$$

$$(ii) \qquad \hat{x}(\alpha), \quad \hat{\lambda}(\alpha), \quad \hat{\mu}(\alpha), \quad \hat{F}(\alpha) \equiv F(\hat{x}(\alpha);\alpha)$$

For a given value of the parameter vector, $\alpha_0 \in A$, assume that $h(x^*(\alpha_0);\alpha_0) = 0$. (That is, the additional constraint in problem (*ii*) is satisfied at problem (*i*)'s solution for $\alpha = \alpha_0$.) Further assume:

$$\frac{\partial g}{\partial x}(x^*(\alpha_0);\alpha_0) \text{ and } \frac{\partial h}{\partial x}(x^*(\alpha_0);\alpha_0) \text{ are linearly independent.}$$

a.) It is obvious that $\hat{x}(\alpha_0) = x^*(\alpha_0)$ and $\hat{F}(\alpha_0) = F^*(\alpha_0)$. Show that:

$$\frac{\partial F^*}{\partial \alpha}(\alpha_0) = \frac{\partial \hat{F}}{\partial \alpha}(\alpha_0).$$

(Hint: Use the FONC for the two problems to argue that $\hat{\mu}(\alpha_0) = 0$. Then use the envelope theorem.)

b.) Prove that

$$\frac{\partial^2 \hat{F}}{\partial \alpha^2}(\alpha_0) - \frac{\partial^2 F^*}{\partial \alpha^2}(\alpha_0) \text{ is negative semi-definite.}$$

(Hint: Argue that $\hat{F}(\alpha) \leq F^*(\alpha)$ for all $\alpha \neq \alpha_0$ and then use a Taylor series expansion.)

(Note: The same result could be established for the more general case in which the numbers of constraints in problems (*i*) and (*ii*) are r_1 and r_2, respectively, where $0 \leq r_1 \leq n-2$ and $r_1 < r_2 < n$, and the additional constraints in problem (*ii*) are satisfied at problem (*i*)'s solution for $\alpha = \alpha_0$.)

2. A profit maximizing firm uses inputs, x_1 and x_2, to produce output, *y*, via the production function $y = f(x_1, x_2)$. The firm faces parametric output and input prices: $p, w_1, and\ w_2$.

a.) In the "long run," the firm can respond to price changes by choosing profit-maximizing input quantities without constraint. In the "short run," the firm is required to maintain the same ratio between x_1 and x_2 that characterizes its current equilibrium.

(For example, suppose that x_1 is unionized labor and x_2 is the number of machines. The union, concerned about a trend toward declining employment, has negotiated a contract mandating that firms hire at least two workers for every machine, for "safety" reasons. In this case, the "short run" corresponds to the period for which the current contract is in force. Actually, this hypothetical is similar to a famous real-world instance of "featherbedding," a labor union practice designed to stimulate employment by imposing inefficient work rules. "The classic example of featherbedding in the United States is the railroad union requirement that railroad owners hire firemen, men who shoveled coal, to work on diesel locomotives, which don't use coal." This effectively required that there be two workers (engineer and fireman) for every machine (diesel locomotive), when only one was needed. http://en.wikipedia.org/wiki/Featherbedding)

Use the result of problem 1 to show that short-run factor demands are no more elastic than long-run factor demands.

b.) As an alternative, suppose that the short-run constraint is that the firm maintains the same total expenditure on factors as in its current equilibrium. Is it true for this case that short-run factor demands are necessarily no more elastic than long-run factor demands? Explain.