Economics 601, Microeconomic Analysis I — Harvey Lapan
Fall 2010

Final Exam - Answer Any Three Questions

1. A monopoly producer (e.g., an electric utility) has the following production function:

$$Q = \theta\left(K \cdot L^{1/2}\right); \quad \theta = \left(3^{3/2}/2\right)$$

K is capital inputs, *L* is labor inputs and factor prices are given by $W = R = 1$. The industry demand for this product is given by:

$$Q^D = AP^{-3/2}$$

a) Find the monopolist's cost function and its profit maximizing price and output. Is this equilibrium efficient? **(9 points)**

b) Find the socially efficient output level. **(7 points)**

i. Assuming private enterprise (the government does not operate the firm), what government policy can be used to achieve the efficient equilibrium? Specify the magnitude of this policy. Is breaking the monopolist up into a number of competitive firms a way to achieve efficiency? **(6 points)**

c) Suppose the only policy the government can use is to set a ceiling on the firm's price. Naturally, in the long run the firm will only produce if its profits are non-negative. Find the optimal long run price ceiling, given this profit constraint. (optimal means the policy maximizes the sum of profits and consumer surplus, subject to the profit constraint). **(7 points)**

i. Suppose investment decisions (the choice of *K*) must be made before labor (and output) decisions are made. Thus, the "long run" is a time period in which both *K* and *L* are variable, whereas in the "short run" only labor is a choice variable (and investment decisions represent fixed costs). Does the government have an incentive to change the price ceiling after investment decisions are made? (only a verbal discussion is needed here) **(4 points)**

2. A young entrepreneur, Sara, with current wealth w_0, is considering investing her own assets in a project that may lead to a successful business (e.g., a new social network site). How much the business is worth after this investment is a random variable, whose expected value is an increasing function of the amount invested. To be concrete, define:

c as the amount invested in the project

$\lambda V(c)$ as the ultimate value of the project, where λ is a random variable; and $V'(c) > 0 > V''(c)$

$\lambda \in \left[0, \lambda^u\right]; \quad E(\lambda) = 1; \quad f(\lambda)$ is the density function for the random variable.

Thus, $V(c)$ is the expected value of the business, and Sara's *ex post* wealth will be: $w = w_0 - c + \lambda V(c)$. Sara has the strictly concave Bernoulli utility function $u(w)$ and, having gotten her Ph.D. in economics, she always behaves as an expected utility maximizer.

a) Set up the expected utility maximization problem and show how Sara determines the optimal level of investment (c). **(8 points)**

   i. How will an increase in w_0 affect Sara's investment decision? Prove your answer. **(7 points)**

   ii. Assume Sara's spouse, Jeff, is more risk averse than she is and that Jeff's Bernoulli utility function is $\sigma(w) = H(u(w)), \quad H' > 0 > H''$. If Jeff, who also is an expected utility maximizer, were to choose how much to invest in this project, how would his choice compare to Sara's? Be specific and prove your answer. **(6 points)**

b) Next, assume there is an active venture capital market and that, once Sara completes her project – but before λ, and hence the project's true value, is known – Sara can sell an interest in the project to these investors. These investors – like Sara – can assess the expected value of the project $V(c)$, and are prepared to buy an interest in the business based upon this expected value. Sara can choose what fraction $s \in [0,1]$ of the business to sell to these investors, and the investors will pay her $(s\theta V(c)), \theta < 1$ for that share (note that the investors are paying less than the expected value). Thus, Sara's *ex post* wealth is: $w = w_0 - c + \lambda V(c)(1-s) + s\theta V(c)$.

   i. Assuming expected utility maximization, and that Sara chooses both c and s before λ is known, show how the values of c and s that maximize expected utility are determined. Will Sara sell her entire interest in the business? **(6 points)**

   ii. How do increases in Sara's initial wealth affect her optimal investment expenditures (c) and the amount of the business she chooses to sell (s)? Be specific. **(6 points)**

3. Consider a competitive industry in long run equilibrium. All firms are identical with the following cost function:

$$C(w_1, w_2, q) = \left(q^{3/2} + 9q^{1/2}\right)\left(w_1^{1/2} w_2^{1/2}\right)$$

where q denotes the firm's output and w_1, w_2 the prices of the two inputs used to produce q. Let $D(p, A) = Ap^{-1/2}$ denote the demand for the industry's output, where p is price and A a demand shift parameter (assume A is large and solutions are interior). The industry long-run equilibrium is characterized by (p^*, q^*, J) where p^* is equilibrium price, q^* is equilibrium output per firm and J is the number of (identical) firms (you may treat J as a continuous variable,).

(a) Assuming input prices are exogenous, derive the long run industry supply curve, find the equilibrium values of (p^*, q^*, J), and show how they change with A. **(9 points)**

   i. Suppose an *ad valorem* tax at rate τ is imposed on output, so that $p^c = (1+\tau)p$, where p^c is the consumer price and p the net producer price. Compare the effects of this tax on: (1)output per firm, (2)firm's profits; (3)equilibrium price; and (4)total industry output in the short run, when the number of firms is fixed, and in the long run. **(7 points)**

(b) Assume that $w_1 = 1$ is exogenous, but that w_2 (the price of input two, z_2) is affected by this industry's demand for that input. Input 2 (z_2) is produced by a competitive profit-maximizing industry with the following (long run) supply curve:

$$z_2^S = S(w_2) = \mathrm{B}w_2 ; \quad \mathrm{B} > 0$$

Recalling that one can get the firm's conditional input demand from the cost function, **derive** the long run industry supply curve for good *q*. (Remember this requires demand equal supply in the market for input 2. If you cannot derive the long run industry supply curve, write down the set of equations which would allow you to derive it). **(9 points)**

i. What does the area next to the long run industry supply curve for good *q* measure? Be specific and, if possible, prove your answer. **(8 points)**

4. Consider a doctor who has her own medical practice. She produces an output, *q*, called medical services using her own time (*L)* and nurses (*N)* to produce output:

$$q = F(L, N)$$

The production technology $(F(L, N))$ is a concave, constant returns to scale production process. The price, p_q, which she charges for medical services, and the price *R* which she pays for nurses, are exogenous. The doctor has exogenous income (w_0) and her (strictly quasi-concave) utility function depends upon her leisure (l) and her consumption of a composite consumption good (*c*), whose price is p_c. She behaves as a utility maximizer, so her problem is to:

$$\text{Max } U(c, l) \text{ subject to: } (w_0 + p_q q - RN - p_c c) \geq 0; \quad (l + L) \leq T = 24; \quad q = F(L, N)$$

Note that *T* denotes her aggregate time endowment.

(a) Write down the first own conditions for this problem and interpret them (assume an interior solution). Will the second order conditions be satisfied? **(8 points)**

(b) Assuming both *c* and l are normal goods, how will an increase in w_0 affect her optimal choices of $(c^*, l^*, L^*, N^*, q^*)$. Be as specific as possible. (Hint: you should be able to tell, from the given assumptions, the sign of $(\partial^2 F / \partial L \partial N)$). **(8 points)**

(c) Suppose both the price of output p_q and the price of nurses R double. Explain how this change will affect the doctor's optimal choices and under what conditions the output of medical services increases (or decreases). Be specific. **(7 points)**

(d) For the special case where $U = (c \cdot l)$ and $q = 2(L \cdot N)^{1/2}$ find the optimal solution. **(10 points)**

5. Answer all parts.

(a) Consider an economy with H consumers, each of whom has the following preferences:

$$U^h = m^h + \alpha_1^h x_1^h + \alpha_2^h x_2^h - \frac{\left(x_1^h\right)^2}{2} - \frac{\left(x_2^h\right)^2}{2} + \gamma x_1^h x_2^h; \quad |\gamma| < 1; \quad \alpha_i^h > 0, i = 1, 2$$

There are three consumption goods – the numeraire (good m), and goods 1 and 2. Let $\left(p_1^c, p_2^c\right)$ be prices facing the consumer, where the price of good m is normalized to 1. There are a large number of identical firms who can produce either good one or good two, using the numeraire good as input. The production costs, measured in terms of the numeraire, for firm j are:

$C^j = \left(y_1^j + y_2^j\right)$, where y_i^j is firm *j's* output of good *i*.

i. Suppose there is no tax on good 2, but an *ad valorem* tax at rate τ_1 on good *1*. Assuming an interior solution for each consumer, and assuming the tax revenue is rebated to consumers, find the deadweight loss from the tax. **(8 points)**

ii. Assume $\tau_1 > 0$ and that the government cannot change this tax. Can a tax (or subsidy) on good 2 increase welfare? First give an intuitive answer, **and then** calculate the optimal tax τ_2. **(9 points)**

(b) Consider the special case of a two input production function: $q = F(K, L)$. Let (R, W) represent input prices, and p denote output price. Assume the firm maximizes (expected) profits, and suppose decisions are made in the following sequence:

(1)the firm chooses K based upon its beliefs about output price
(2)the firm learns the true output price
(3)the firm chooses L to maximize profits

The long run supply curve refers to the situation in which both inputs are variable, whereas the short run supply curve refers to the situation in which only L is a choice variable.

i. Compare the impact of a price increase on output and on labor (L) demand in the short run and the long run. Relate your answer to specific properties of the production function (i.e., do not just say you cannot compare two entities – say what you need to know to be able to compare them). **(6 points)**

ii. Consider an environment of true uncertainty. Suppose that **price *p* is not known when *K* is chosen,** but its distribution $F(p)$, and hence its expected value $E(p) = \int p dF(p)$ is known. **Price is known when *L* is chosen**. **Assume the firm is risk neutral and thus maximizes expected profits.** Given the production function, $q = \left(K^{1/4} L^{1/2}\right)$, find the values of K and L that maximize expected profits. Discuss how to relate this approach to the idea of short run and long run supply curves. **(10 points)**