Economics 601, Microeconomic Analysis I — Harvey Lapan
Fall 2010

Midterm 2
Answer Any Three Questions

1. Answer all parts.

(a) Consider a utility maximizing consumer with monotonic and quasiconcave utility function $u(x)$. Prices and income currently facing the consumer are $(\vec{p}^0, w^0)$, but may change (e.g., through government policy) to the price-income vector $(\vec{p}^1, w^0)$.

i. Define the compensating variation $CV(\vec{p}^0, \vec{p}^1, w^0)$ and equivalent variation $EV(\vec{p}^0, \vec{p}^1, w^0)$ to this price change. Is it true that $CV(\vec{p}^0, \vec{p}^1, w^0) > 0 \Leftrightarrow EV(\vec{p}^0, \vec{p}^1, w^0) > 0$? **(5 points)**

ii. A different policy the government is considering would result in the price-income vector $(\vec{p}^2, w^0)$ and the government is debating which policy to implement. If it finds $CV(\vec{p}^0, \vec{p}^2, w^0) > CV(\vec{p}^0, \vec{p}^1, w^0) > 0$, does this imply $EV(\vec{p}^0, \vec{p}^2, w^0) > EV(\vec{p}^0, \vec{p}^1, w^0) > 0$? Prove your answer; if you can't, give an intuitive explanation. (Hint: Consider quasi-linear preferences with two goods). **(7 points)**

iii. Does your answer to part (ii) change if you know preferences are homothetic? Prove your answer. **(5 points)**

(b) An individual's utility depends upon her consumption of a (composite) good (x) and leisure (l). She is endowed with $T = 24$ units of time, which is divided between leisure and work (*L*). The price of the good, and the wage rate, are initially (p^0, W^0), and she receives an exogenous transfer *I* (if *I<0* she pays this sum) so that her full income is $Y^f = W^0 T + I$. Her Marshallian and Hicksian demands for leisure are: $l^M = \frac{2Y^f}{3W}$; $l^H = \frac{2U}{3}\left(\frac{p}{W}\right)^{1/3}$ (where *U* is utility). Initially, $(p^0 = 1, W^0 = 8, I = 0)$.

i. She has the opportunity to obtain a job that pays a wage of $W^1 = 27$, but getting this job requires her to incur a fixed cost of *F* (the price of *x* is unchanged). What is the maximum amount (maximum value of *F*) she would spend to get the job? **(8 points)**

ii. Suppose the new job requires her to work at least 16 hours a day (a graduate student!) at wage $W^1 = 27$. Now what is the maximum amount she would pay to get the job? **(8 points)**

2. A firm, which produces an output q, using two inputs (z_1, z_2), has the following supply and factor demand curves: $q^* = \frac{10p}{(R_1 R_2)^{1/2}}$; $z_1^* = \frac{\lambda p^\alpha}{R_1^\beta R_2^{1/2}}$, $z_2^* = \frac{\lambda p^\alpha}{R_1^{1/2} R_2^\beta}$, where p is output price, (R_1, R_2) are factor prices, and (λ, α, β) are parameters to be determined.

a) Find the values of (λ, α, β) predicted by theory. **(6 points)**

b) Calculate the change in the firm's profits if the price vector it faces changes from $(p, R_1, R_2) = (1,1,1)$ to $(p, R_1, R_2) = (4,9,1)$. (If you could not get an answer to part i, you may express your answer to this part in terms of the parameters). **(6 points)**

c) Find the production function dual to the supply and factor demands and derive the conditional factor demands $\left(z_i^c(q, R_1, R_2)\right)$. **(15 points)**

d) The firm is offered access to a technology that doubles its productivity (*i.e.*, if the firm's production function derived in part (c) is $q = f(z_1, z_2)$, the new technology would transform this to $q = 2f(z_1, z_2)$). Given the price vector $(p, R_1, R_2) = (4,9,1)$, find the maximum amount the firm would pay to acquire this technology and also its profit-maxmizing output if it acquires the new technology. **(6 points)**

3. Answer all parts.

a) Consider a multi-product firm which produces two outputs (q_1, q_2) using two inputs (z_1, z_2). Suppose its production technology is given by:
$\left(q_1^2 + q_2^2\right) \le \left(z_1^\alpha + z_2^\alpha\right)^{\beta/\alpha}$; $\beta > 0$; $q_i \ge 0, z_i \ge 0$, $i = 1,2$
Define the production set $Y = \left\{(q_1, q_2, -z_1, -z_2) \in R^4 \quad s.t. \quad \left(q_1^2 + q_2^2\right) \le \left(z_1^\alpha + z_2^\alpha\right)^{\beta/\alpha}\right\}$

   i. What conditions on (α, β) imply that Y is a convex set? Why does convexity of the set matter? **(5 points)**

   ii. Let prices of goods be (p_1, p_2) and prices of inputs be (R_1, R_2) so the firm's profits are: $\pi = \sum_{i=1}^{2} p_i q_i - \sum_{i=1}^{2} R_i z_i$. For $\alpha = 2$, is Y convex? What values can $\beta \ge 0$ assume if there is to be a solution to the profit maximizing problem? **(5 points)**

   iii. Assume $\alpha = 2, \beta = 1$. Find the firm's profit-maximizing decisions and maximum profit function. **(10 points)**

(***question 3, part b on next page)***

b) Consider an individual who consumes L goods and whose preferences are represented by a strictly increasing, strictly quasi-concave utility function that exhibits separability as indicated below:

$$U\left(x_1, x_2, ..., x_L\right) = U\left(\phi\left(x_1, ..., x_m\right), \theta\left(x_{m+1}, ..., x_L\right)\right)$$

Let X^A denote the first set of goods $\left(x_1, ..., x_m\right)$, and let $\vec{P}^A$ denote the price vector of these m goods. Similarly, let X^B denote the second set of goods $\left(x_{m+1}, ..., x_L\right)$, and let $\vec{P}^B$ denote the price vector of these $\left(L-m\right)$ goods. The functions $\phi(...)$ and $\theta(...)$ are strictly increasing, concave and homogenous of degree one functions, and U is a homothetic function. Finally, assume the individual maximizes utility subject to the budget constraint: $w \geq \sum_{i=1}^{L} p_i x_i$.

i. What does this structure imply about the demand functions for goods? For example, does the demand for goods in X^A depend upon the prices of goods in X^B? Does the ratio of demands for goods in X^A depend upon the prices of goods in X^B (e.g., does the ratio of demands $\left(x_1/x_2\right)$ depend upon p_L)? Be as specific as possible about what you can infer about the demands. **(8 points)**

ii. If utility has the specific form $U\left(x_1, x_2, ..., x_L\right) = \phi\left(x_1, ..., x_m\right) \cdot \theta\left(x_{m+1}, ..., x_L\right)$, what more can you infer about the demands? Be specific. **(5 points)**

4. Answer All Parts

(a) Consider a competitive profit-maximizing firm that produces an output, q, using three inputs $\left(z_1, z_2, z_3\right)$. Output and input prices are given by $\left(p, R_1, R_2, R_3\right)$ and the firm's strictly concave production function by $f\left(z_1, z_2, z_3\right)$. In the short run, z_3 is fixed at $\overline{z}_3$, while $\left(z_1, z_2\right)$ are variable inputs, whereas in the long run all inputs are variable. Let $\pi^s\left(p, R_1, R_2, R_3, \overline{z}_3\right)$ denote the short run (restricted) profit function, and $\pi^L\left(p, R_1, R_2, R_3\right)$ the long run profit function. Similarly, let $\left(\hat{q}^s, \hat{z}_1^s, \hat{z}_2^s\right)$ denote the short run profit-maximizing choices of the firm and $\left(\hat{q}^L, \hat{z}_1^L, \hat{z}_2^L, \hat{z}_3^L\right)$ the long run profit maximizing choices. Let $\hat{P}\left(\overline{z}_3\right) = \left\{\left(p, R_1, R_2, R_3\right) \in \Re_+^4\right\}$ such that $\hat{z}_3^L\left(p, R_1, R_2, R_3\right) = \overline{z}_3$ for $\left(p, R_1, R_2, R_3\right) \in \hat{P}\left(\overline{z}_3\right)$.

i. What can you conclude about the sign of $\left(\hat{q}^L - \hat{q}^s\right)$? What can you conclude about the sign of $\left(\frac{\partial \hat{q}^L}{\partial p} - \frac{\partial \hat{q}^s}{\partial p}\right)$? Be as specific as possible. (Naturally, the short and long run variables are evaluated at the same points in price space) **(6 points)**

ii. "An increase in R_1 will reduce the profit-maximizing output in the short run, but it will reduce output even more in the long run." Evaluate this statement. **(6 points)**

(*question 4, part b on next page)*

(b) A production function $f\left(\vec{x}\right)$ exhibits increasing returns to scale if $f\left(t\vec{x}\right) > tf\left(\vec{x}\right)\ \forall t > 1, \forall \vec{x} > 0$, and decreasing returns to scale if $f\left(t\vec{x}\right) < tf\left(\vec{x}\right)\ \forall t > 1, \forall \vec{x} > 0$ (note that Jehle's definition and MGW's definition are not identical but they have the same implication).

i. What does increasing returns to scale imply about how average cost and marginal cost change as output increases? What does decreasing returns to scale imply about how average cost and marginal cost change as output increases? Be as specific as possible. **(6 points)**

ii. Will there be a profit-maximizing solution for a competitive firm if the production function exhibits increasing returns to scale? Explain. **(3 points)**

iii. Suppose a firm has the production function: $q\left(x_1, x_2\right) = h\left(z\left(x_1, x_2\right)\right)$, where:

$$h\left(z\right) = z\left(1 - e^{-z}\right), \quad z\left(x_1, x_2\right) = \left(x_1^{-1} + x_2^{-1}\right)^{-1}; \quad x_1 > 0, x_2 > 0$$

Given factor prices $\left(R_1, R_2\right)$, find the cost curve dual to this production function. {Note that *h(..)* is a positive monotonic transformation of *z*. Your answer for the cost function may be expressed in terms of this transformation, or its inverse, as appropriate). **(7 points)**

iv. Does the production function from part (iii) exhibit increasing returns to scale? What can you conclude about the slopes of the average and marginal cost curves? Be specific. **(5 points)**