Economics 601 Harvey Lapan

Microeconomic Analysis I Fall 2010

Problem Set No. 1 Due by: Friday, August 27

1.1. Graph the sets A and B below and decide whether or not they are convex.

$$A = \left\{ (x, y) : x \geq 0, y \geq 0, \quad 2x + y + Max\left[(y-5), 0\right] \leq w \right\}; \quad w > 5, \text{ a scalar}$$

$$B = \left\{ (x, y) : x \geq 0, y \geq 0, \quad 2x - Max\left[(y-5), 0\right] + 2y \leq w \right\}; \quad w > 10 \text{, a scalar}$$

NOTE: $Max\left[(y-5), 0\right] = 0, \quad y \leq 5; \quad Max\left[(y-5,)0\right] = (y-5) \quad for \quad y \geq 5$

1.2. Set A in problem set 1.1 can represent a budget set for someone who has income *w* and faces prices $(p_x, p_y) = (2,1)$ if he buys no more than 5 units of good *y*, but must pay a price of $p_y = 2$ for all units of *y* purchased in excess of 5. Similarly, set B can be thought of as a similar case, **except** that the price of *y* is *2* for the first 5 units of *y*, whereas the price declines to $p_y = 1$ on all units of *y* purchased in excess of 5. Suppose you want to maximize the function: $U = xy$ **.**

a. Find the maximum for (*x*,*y*) in set A and relate your answer to the value of *w*. Can you be sure a local maximum is a global maximum?

b. What difficulties do you encounter in finding a maximum for (x,y) in set *B*? Is every local maximum going to be a global maximum?

c. Find the optimum for *(x,y)* in set B and relate your answer to the value of *w*.

1.3. Consider the function $f : \mathbb{R}^2_{++} \to \mathbb{R}$ specified as $f(x_1, x_2) = \left[x_1^\alpha + x_2^\alpha\right]^{(\beta/\alpha)}, \quad \lambda > 0$.

a. Derive restrictions on α and β which ensure that f is concave.

b. Derive restrictions on α and β which ensure that f is quasiconcave.

c. Derive restrictions on α and β which ensure that f is convex.

1.4 Suppose we have two functions $f, g : \mathbb{R}^2_{++} \to \mathbb{R}$: $f(x_1, x_2) = x_1^\alpha x_2^\beta$, $g(x_1, x_2) = x_1^\eta x_2^\phi$. Assume all parameters $(\alpha, \beta, \eta, \phi)$ are positive.

a. If (α, β) and (η, ϕ) are such that *f* and *g* are concave, is $(f + g)$ necessarily concave?

b. If (α, β) and (η, ϕ) are such that *f* and *g* are convex, is $(f + g)$ necessarily convex?

c. If (α, β) and (η, ϕ) are such that *f* and *g* are quasi-concave, is $(f + g)$ necessarily quasi-concave? Explain your answer to each part.

1.5. Let $f : [-10, 20] \to \mathbb{R}$ be given by $f(x) = x^3 - 27x$.

Use first and second order conditions to identify local minimum and maximum points, and argue why each of these local maximum (minimum) need not be a global maximum (minimum).

If the domain were $[-10, 4]$ would your answer concerning the relationship between the local and global extreme point(s) change? How?

1.6. For the following three versions of the function $f: S \to \mathbb{R}$ discuss whether Weierstrass' theorem applies and indicate the global maximum and minimum (if they exist):

(i) $S = \mathbb{R}$ and $f(x) = x^4$

(ii) $S = (0,1)$ and $f(x) = x$

(iii) $S = [-4,4]$ and $f(x) = \begin{cases} \dfrac{x^3 - 8}{x-2}, & x \neq 2 \\ a & x = 2 \end{cases}$, a a scalar.

1.7 Let $f(x_1, x_2)$ be a concave function, and assume $(x_1, x_2) \in D \subset R^2$.

a. Must a global maximum exist in D for $f(x_1, x_2)$?

b. If we find (x_1^*, x_2^*) is a local maximum, can we conclude it is a global maximum?

c. Define a new function: $g(x_1, x_2) \equiv H(f(x_1, x_2))$ where $H: R \to R$. Suppose H is differentiable and $\dfrac{dH}{df} > 0$ everywhere. Is the function $g(x_1, x_2)$ necessarily concave?

d. Same assumptions as (b) and (c). Will (x_1^*, x_2^*) be a local maximum for $g(x_1, x_2)$? Will it be a global maximum? Explain your answer.