Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2010

Problem Set No. 8 Due by: Thursday, October 28

1. Suppose inputs $(z_1,...,z_n)$ are used to produce good q according to the following production function: $q \le f(z_1,...,z_n)$. Let factor prices $(W_1,...,W_n) \gg 0$ and output price $p > 0$.

   (a) Set up the profit maximizing function and write the profit maximizing conditions for an interior solution. What are the second order sufficient conditions (SOSC)?

   (b) Set up the cost mimimizing problem, and write the first order conditions for deriving the cost function $c(\vec{w}, q)$ for this same production function. What are the SOSC?

   (c) Consider a positive monotonic transformation $g(z_1,...,z_n) = H(f(z_1,...,z_n))$, $H(0)=0$, $H' > 0$. How does this transformation of technology change the cost-minimizing factor demands and the cost function? How does it change the profit-maximizing factor demands and output supply?

2. Consider a firm which uses N inputs to produce one output: $q = f(z_1, z_2,..., z_N)$. Suppose the production function is **concave and homothetic**. In answering, be as specific as possible.

   (a) What does the assumption of homotheticity imply about:
   (i) the cost-minimizing (conditional) factor demands?
   (ii) the profit maximizing output supply and input demand curves?
   (iii)the cost function and the profit function?

   (b) Are inferior inputs possible with homothetic technology? Explain why and use your results from (1c) to show how a positive monotonic transformation of a concave, homothetic production function alters the: (i)cost function; (ii)cost-minimizing factor demands; (iii)profit function; and (iv)profit-maximizing input demands. **Be as specific as possible**.

   (c) Illustrate parts (a)-(b), by finding the cost minimizing solution and the profit maximizing solution (if it exists) for:

   $$f(z_1,z_2) = \left[z_1^{\rho} + z_2^{\rho}\right]^{1/(2\rho)}; \; g(z_1,z_2) = H(f) = f^{\mu} = \left[z_1^{\rho} + z_2^{\rho}\right]^{\mu/(2\rho)}; \; \rho < 1, \; \rho \ne 0; \; \mu > 0$$

3. A firm that uses two inputs to produce one output has the following production function:

   $q = \left\{Min\left[(x_1 - A), 0\right] \cdot x_2\right\}^{1/4}$; $\quad x_1 \ge 0, x_2 \ge 0$ (to be clear, q=0 if $x_1 \le A$)

   (a) Is the firm's production set convex? Does it exhibit (global) decreasing returns to scale?

   (b) Find the firm's cost function, profit-maximizing input demands and output supply, and its maximum profit function.

4. (Short Run vs. Long Run Cost Functions) Consider a single product firm with the following technology:

$$q = f\left(z_A, z_B\right), \quad z_A = \mathbb{R}_+^N, \quad z_B = \mathbb{R}_+^M$$

Let the corresponding factor price vectors be w_A, w_B and define the short run and long run costs as:

$$C^s\left(q; w_A, w_B, \overline{z}_B\right) = \underset{z_A}{Min}\left(w_A \cdot z_A + w_B \cdot \overline{z}_B\right) \quad f\left(z_A, \overline{z}_B\right) \geq q$$

$$C^L\left(q; w_A, w_B\right) = \underset{z_A, z_B}{Min}\left(w_A \cdot z_A + w_B \cdot z_B\right) \quad f\left(z_A, z_B\right) \geq q$$

In words, the vector of inputs, z_B is fixed in the short run, but variable in the long run. Let $z_A^L\left(q; w_A, w_B\right), z_B^L\left(q; w_A, w_B\right)$ denote the long run conditional factor demands, and suppose there exists $\left(\overline{q}, \overline{w}_A, \overline{w}_B\right) \, s.t. \, z_B^L\left(\overline{q}, \overline{w}_A, \overline{w}_B\right) = \overline{z}_B$.

(a) Compare short run and long run costs, and short run and long run conditional factor demands, when evaluated at $\left(\overline{q}, \overline{w}_A, \overline{w}_B\right)$. Also, compare the values of short run and long run marginal costs, and the slopes of the marginal cost curves at $\left(\overline{q}, \overline{w}_A, \overline{w}_B\right)$.

(b) Let w_A^i denote the price of the i^{th} input in set *A* (so that it is variable in both the short run and long run). Compare how an increase in w_A^1 affects the short run and long run conditional factor demands for inputs in set *A*, and compare the impact of this price increase on short run and long run marginal cost (as always, evaluate this change at $\left(\overline{q}, \overline{w}_A, \overline{w}_B\right)$).

(c) Use your result from previous parts to compare the slopes (with respect to output price) of the long run and short run profit maximizing output supply curves, and to compare the slopes (with respect to own input price) of the short run and long run input demand curves. **Do your predictions about the relative values of these slopes hold away from the vector** $\left(\overline{q}, \overline{w}_A, \overline{w}_B\right)$?

(d) Consider the production technology: $q = \left(x_1 x_2\right)^{1/2} + x_3^{1/2}$. Assuming x_2 is fixed in the short run, find the short run and long run cost curves and conditional factor demands. Also, find the short run and long run profit maximizing output supply and factor demands (or, if no solution exists, indicate why).

5. For the following cost functions, find the (dual) production functions:

(a) $C\left(q, w_1, w_2\right) = \left(w_1 w_2\right)^{1/2} q + w_2 q^2$

(b) $C\left(q, w_1, w_2\right) = \left(w_1^{\alpha} w_2^{1-\alpha}\right) q^2; \quad \alpha = (0,1)$

(c) $C\left(q, w_1, w_2\right) = \begin{cases} w_1\left(q/2A\right)^2 & for \quad q \leq \left[2A^2\left(w_2/w_1\right)\right] \\ w_2 q - \left(A^2 w_2^2/w_1\right) & for \quad q \geq \left[2A^2\left(w_2/w_1\right)\right] \end{cases}$

6. {Peak Load Problem} A utility must meet output targets in two periods. Its production technology is given by $q_t = \left(x_{1,t} \cdot x_{2,t}\right)^{1/2}$ where *t=1,2* indexes time. The price for the inputs are the same in each period (and given by w_1, w_2). Suppose the output targets are $q_2 = \lambda q_1; \quad \lambda > 1$

   (a) Assuming inputs can be chosen separately for each period, find the cost-minimizing solutions and total cost for the two periods.
   (b) Suppose that input one (e.g., labor) can be freely varied between periods but input two (e.g., capital) must be the same for the two periods. Set up the cost minimizing solution and solve.
   (c) Given total output, *Q* {so that $\left(q_1 + q_2\right) = \left(1+\lambda\right) q_1 = Q$ } for the two periods, show how increases in λ affects total costs for case (i) and for case (ii). Discuss the meaning of this result.

7. Let $q = \left(x_1 + x_2^2\right)$

   (a) Is this production function concave? Is it quasi-concave?
   (b) Find the cost curve for this production function and find the profit-maximizing solution, if one exists.
   (c) Find the production function which is dual to the cost curve from part (b). Will it be the same as the production function you started with? Why or why not?
   (d) Suppose $q = \left(x_1 + x_2^2\right)^{1/4}$. Is this function concave or quasi-concave? Find the cost curve and profit-maximizing solution for this function.

8. There are two firms (*A,B*). Each firm uses inputs of z_1 and z_2 to produce output of good *q*. Let $\left\{z_1^i, z_2^i, q^i\right\}$ denote firm *i's* input and output vector and assume they have the following technology:

$$q_1^A \le 10\left(x_1^A \cdot x_2^A\right)^{1/4}; \quad q_1^B \le 5\left(x_1^B \cdot x_2^B\right)^{1/4}$$

   (a) Find each firm's profit maximizing decisions and its maximum profit function, $\pi^i\left(p, w_1, w_2\right)$. Also, find the "aggregate" supply and factor demand curves by adding together the supply and demand curves for the two firms.

   (b) **Next**, derive the "industry" production function, $q^T\left(x_1^T, x_2^T\right)$ by solving the following problem: Given $x_1^T, x_2^T, \quad Max\left(q_1^A + q_1^B\right)$ subject to the resource constraints: $x_i^A + x_i^B \le x_i^T, \; i = 1,2.$

   (c) **Find the** aggregate supply and input demands $\left(x_1^T, x_2^T, q^T\right)$ that maximize profits for the aggregate production function derived in part (b), and find the maximum (aggregate) profits.

   (d) **Show** that the aggregate maximum profit function, and the corresponding output supply and input demands, are just the sum of the individual firm's rules (e.g., $\pi^T = \pi^A + \pi^B$).

   (e) An aggregate netput vector y^T is said to be efficient if there does not exist a technological feasible vector y' (in the aggregate production set) such that $y' \ge y^T, \quad y' \ne y^T$ {in this case, the netput vector is $y = \left\{-x_1, -x_2, q\right\}$ }. Use the previous parts to argue that individual profit maximization leads to an aggregate production vector that is efficient.