Economics 601 Harvey Lapan
**Microeconomic Analysis I** Fall 2010

Problem Set No. 10 Due by: Thursday, November 19

1. Do Question 5, Problem Set 9

2. (Market failure, NOT covered in class). Consider a simple model with H identical consumers, 2 produced good plus the numeraire, and 2 types of firms (H of each type), each producing one good using the numeraire as input.

   Consumer Preferences: $U^h = m^h + A\left(\ln c_1^h + \ln c_2^h\right) - \phi z; \quad h = 1,...,H; \quad z = \sum_{j=1}^{H} q_2^j$

   Firms producing good 1: $c_1^j\left(q_1^j\right) = \left(\left(q_1^j\right)^2 / 2\right); \quad j = 1,...,H$

   Firms producing good 2: $c_2^j\left(q_2^j\right) = \left(\left(q_2^j\right)^2 / 2\right); \quad j = 1,...,H$

   Resource constraints: $\sum_j q_i^j \geq \sum_i c_i^h, \quad i = 1,2; \quad M^T - \sum_1 c_1^j - \sum_2 c_2^j - \sum_h m^h \geq 0$

   Everything is standard **except** the "z" in the consumer's utility function. "z" represents pollution, which is caused by producers of good 2, and which harms consumers if $\phi > 0$. Assume an interior solution for all goods.

   (a) Find the competitive equilibrium. Will it be efficient if $\phi = 0$? Will it be efficient if $\phi > 0$?

   (b) Assuming $\phi > 0$ what policy does the government need to implement to make the competitive equilibrium efficient? (You do not need to solve for the policy, just discuss).

   (c) If the only feasible policy is to tax or subsidize good 1, can such a policy improve welfare? Explain. Would you answer change if the utility function were modified to

   $$U^h = m^h + A\left(c_1^h\right)^{1/4}\left(c_2^h\right)^{1/4} - \phi z; \quad h = 1,...,H; \quad z = \sum_{j=1}^{H} q_2^j .$$

   If so, how? (Again, just a discussion is expected).

3. Let $c(w,q)$ denote the cost function of a competitive firm, where $q$ is output and $w$ is the vector of input prices. Assume that it takes the following form:

   $$c(w,q) = \begin{cases} 4w_1 + q^2\sqrt{w_1 w_2} & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

   (a) Find the firm's profit-maximizing supply function. Now suppose that $w_1 = w_2 = 1$, and let $p$ represent output price. What is the firm's optimal output if $p$ = 6? What about when $p$ = 3?

   (b) Assume that there is free entry in this competitive market, and that input prices are $w_1 = w_2 = 1$. What is the long-run supply correspondence for this industry?

   (c) The output of this industry is demanded by 1,000 consumers, each with indirect utility function $V_i = \omega_i - p + p^2/10$ , where $i$ indexes consumers and $\omega$ denotes income (measured in units of a numeraire good). Input prices are still assumed fixed at $w_1 = w_2 = 1$. Determine the long-run equilibrium (including the long-run number of firms) in this market.

4. Consider a competitive industry in long run equilibrium. All firms are identical, each with cost function $C(w,q)$, where $q$ denotes the output of one firm, and $w$ is the vector of input prices. This cost function displays a U-shaped average cost and a strictly increasing marginal cost. The (downward sloping) market demand for this industry is written as $x(p.\alpha)$, where $p$ denotes the price for the industry output and $\alpha$ is a shift parameter.

   All input prices **except $w_1$ are exogenous.** However, this industry is the only user of input 1, and the market supply of this input is given by $S\left(w_1\right)$, where $S'\left(w_1\right) > 0$.

   The industry long-run equilibrium is characterized by the values of $\left\{p^*, q^*, J^*, w_1^*\right\}$, where $J$ denotes the number of firms. [Strictly speaking $J$ is an integer, but you can ignore that and treat $J$ as a real number].

   (a) Write down the system of equations that define the long-run equilibrium. Briefly discuss the rationale behind each of the equations. Also, show how an increase in demand (an increase in $\alpha$, since $\left(\partial x/\partial\alpha\right) > 0$) affects the equilibrium.

   (b) Now assume $w_1$ is exogenous (i.e., $S\left(w_1\right)$ is infinitely elastic). Use comparative statics on the system of equations derived in (a) – with $w_1$ constant - to determine the impact on the long run equilibrium of an increase in an input price $w_k$, assume input *k* is an inferior input [Recall that an input is said to be inferior if the cost-minimizing input demand is negatively related to output, i.e., $\partial x_k^c(w,q)/\partial q \le 0$]. Specifically, determine the signs of $\partial p^*/\partial w_k$, $\partial q^*/\partial w_k$ and $\partial J^*/\partial w_k$.

5. Consider a model with 3 consumers and one firm (a monopolist) producing good *q*. Assume the monopolist has 3 plants. The consumer's preferences, and the cost function for these plants, are:

   $$\textit{Person } h: \quad u^h = m_h + \alpha^h \left(x_h\right)^{1/2}; \quad \alpha^1 = 3; \quad \alpha^2 = 2; \quad \alpha^3 = 1$$
   $$\textit{Plant } j: \quad c_j\left(q_j\right) = A_j q_j^2; \quad A_1 = 4; \quad A_2 = 2; \quad A_3 = 1$$

   (a) Suppose the monopolist must charge all consumers the same price, Using the aggregate demand curves $D\left(p\right)$, **find the monopoly solution**. The monopolist chooses $q_j, Q, p$ s.t.

   $$\underset{p,Q,q_j}{Max}\left[pQ - \sum_j c_j\left(q_j\right)\right]; \quad q_j \ge Q; \quad Q \le D\left(p\right)$$

   i. Compare the monopoly solution to the efficient solution (which is the competitive equilibrium). What is the deadweight loss due to the monopoly?
   ii. For the given output level, does the monopolist minimize costs?
   iii. What policy – or policies – could the government implement to improve efficiency, given the presence of the monopoly?