Economics 601 Harvey Lapan
**Microeconomic Analysis I** Fall 2010

Problem Set No. 11 Due by: Thursday, December 2, 2010

1. Finish Problem #5 from Problem Set #10 (if not already done)

2. [Monopoly and Price Regulation]. Consider a good, *y*, produced by a monopolist. Consumer preferences for *y* are given by the quasi-linear utility function:

$$U = m + 2Aq^{1/2}y^{1/2}$$

where *m* is the numeraire good. "*q*" denotes the quality of the good, and *y* the quantity of the good consumed (all goods are of the same quality). Assume "income" is large enough so the solution is always interior.

(a) Find the consumer's demand for good *y* as a function of price and quality. Also, express the inverse demand $p(y;q)$ as a function of quality and quantity.

Next suppose the good can be produced with the following cost function:

$$C(q,y) = \left(1+q^2\right)y; \quad q \geq 0; \quad y \geq 0$$

(b) Find the socially optimal quantity and quality of the good (i.e., $\underset{q,y}{Max}\left\{m + 2A(qy)^{1/2} - \left(1+q^2\right)y\right\}$

(c) Suppose quality is fixed at $q = 1$. Find the profit-maximizing output of the monopolist (given the fixed quality) and compare to the socially optimal level.

(d) Assume the government establishes a price ceiling $\bar{P}$ on the price the monopolist can charge. Given the price ceiling, the monopolist chooses price and output $(p, y)$ to maximize profits subject to the constraints: $p \leq \bar{P}$ and $y \leq D(p;q)$ where $D(p,q)$ is the demand for the monopolist's output as a function of price and quality. **Throughout assume quality is fixed at q=1.**

i. Find the monopolist's profit maximizing solution $y^M(\bar{P},q),\ p^M(\bar{P},q)$ at $q = 1$

ii. What happens to $y^M$ as $\bar{P}$ increases? Does the price control raise or lower economic welfare?

(e) Next, assume that the monopolist is free to choose both ***quality*** **and** ***quantity***. **Find the profit-maximizing solution** $y^M, q^M$ (there are no price controls).

(f) Finally, assume the monopolist chooses both quality and quantity, and the government establishes a price ceiling $\bar{P}$. Because the government cannot objectively measure quality, **this price ceiling does not depend on *q*.** Given the price ceiling, the monopolist chooses $\{p, y, q\}$ to maximize profits, subject to the constraints: $p \leq \bar{P}$ and $y \leq D(p,q)$ (note that demand for output depends on quality).

i. Find the monopolist's profit maximizing solution $\tilde{y}^M(\bar{P}),\ \tilde{p}^M(\bar{P}),\ \tilde{q}^M(\bar{P})$.

**ii.** What happens to $y^M$ and $q^M$ as $\bar{P}$ increases? Does the price control raise or lower economic welfare? Contrast your results to part (d).

3. (Tax Incidence and Monopolist)

(a) Consider a monopolist facing inverse demand $p = f(x), \quad f' < 0$, who produces output at constant cost *c*. The government levies a specific tax, *t*, on the monopolist so monopoly profits are given by: $\pi(x) = (p-c-t)x = (f(x)-c-t)x$. Naturally, the monopolist chooses *x* to maximize profits.

i. Can the tax ever result in the monopolist *lowering* price (i.e., can we find $(dp/dt) < 0$)?

ii. Can we find $(dp/dt) > 1$ (i.e., will the price ever increase by more than the tax)?

iii. Assuming demand is linear $(p = a - bx)$ find $(dp/dt)$.

iv. Assuming demand is isoelastic $(p = Ax^{-\alpha})$ find $(dp/dt)$.

(b) For any given specific tax *t*, show that an *ad valorem* tax which results in the same output will lead to more tax revenue.

4. (Third Degree Price Discrimination) Suppose there are two separated markets, with demand in each market given by $x_i = f_i(p_i), \quad i = 1,2$. Under pure monopoly the firm must charge the same price in each market $(p_1 = p_2 = p)$, whereas under price discrimination the prices may vary across markets. Consider a monopolist with constant marginal costs, so profits are: $\pi = \sum_i (p_i - c)x_i, \quad x_i = f_i(p_i)$

(a) Will the monopolist's profits be higher under price discrimination or pure monopoly? Why?
(b) How does price discrimination (as compared to pure monopoly) affect: (i)overall monopoly output; and (ii)overall economic efficiency? Explain your answer.
(c) Answer part (b) for the special case of linear demands $x_i = a_i - b_i p_i$ (assume an interior solution).

5. The Expected Utility (EU) property is cardinal in the sense that it is not preserved under an arbitrary monotonic transformation. On the other hand, this property is preserved under an increasing linear transformation. That is, suppose that $U(L)$ represents an agent's preferences over lotteries and possesses the EU property. Define $\tilde{U}(L) \equiv \kappa + \gamma U(L)$, where $\gamma > 0$. Then, prove that:

(a) $\tilde{U}(L)$ provides the same ranking over lotteries as $U(L)$ does.
(b) $\tilde{U}(L)$ has the expected utility property.

**6.** [INSURANCE] Individual *A*, with Bernoulli utility function $u^A(w)$ has initial wealth $w_0$. With probability $\pi$ *A* will face a loss *L*, while with probability $(1-\pi)$ no loss will occur. A can buy insurance that will pay *I* if the loss occurs, and will pay nothing otherwise. The insurance costs $q$ per dollar of insurance purchased, so the cost of the policy is $qI$. Thus, ex post wealth is as follows:

If a loss occurs: $w_L = w_0 - L + I(1-q)$; If no loss: $w_N = w_0 - qI$

A's problem is to choose *I* to maximize expected utility.

**(a)** Set up the optimization problem and derive the FOC. Are the SOC satisfied?
**(b)** Under what condition will A buy full insurance $(I = L)$?
**(c)** Suppose $q > \pi$. Under what condition will A buy some insurance?
**(d)** Suppose $q > \pi$ and that *A* buys some insurance. Suppose *A* has a friend, *B*, who has the same wealth, faces the same risk of loss *L*, and can buy insurance under the same conditions. *B's* utility function is $u^B(w) = (u^A(w))^{1/2}$. What can you conclude about *B*'s purchase of insurance, as compared to *A*? Be as specific as possible.
**(e)** Suppose *A* has a third friend, *C*, who has the same income, faces the same loss and can buy from the same insurance policy. *C's* Bernoulli utility function is: $u^c(w) = \alpha + \beta u^A(w); \quad \beta > 0$. How will *C*'s insurance decision compare to *A*'s decision?
**(f)** Finally, suppose $u^A(w) = \ln(w)$. Find *A*'s optimal insurance decision, assuming $q > \pi$. Show how $I$ changes as *q* increases; as $w_0$ increases. What happens as *L* increases? (Is $(\partial I^*/\partial L) < 1$?)