Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2010

Problem Set No. 9 Due by: Thursday, November 4, 2010

1. Do problem #8 from Problem Set No. 8.

2. Problem 10.C.3 in MWG textbook (p. 344).

3. (Generalization of Problem 2 above): There are *J* firms, each of which produces the same good (q) using the same set of inputs $\left(z_1,\ldots,z_L\right)$. The production set for each firm is strictly concave and given by:

$$q_j \le F^j\left(z_1^j,\ldots,z_L^j\right)$$

(a) A central planner, with given amounts of each input, wishes to maximize output of good q; the optimization problem is written as:

$$L=\sum_j q_j+\sum_l \gamma_l\left(z_l^T-\sum_j z_l^j\right)=\sum_j F^j\left(z_1^j,\ldots,z_L^j\right)+\sum_l \gamma_l\left(z_l^T-\sum_j z_l^j\right)$$

where z_l^T is the availability of each input. The Lagrangean, evaluated at the solution, gives the industry production function: $Q^*\left(z_1^T,\ldots z_L^T\right)=L\left(z_1^T,..z_L^T;z_l^{*j},\gamma_l^*\right)$; $\left(z_l^{*j},\gamma_l^*\right)$ is the solution vector.

i. Write the FOC for the optimizing problem and interpret. Are the SOSC satisfied?
ii. What is the meaning of the Lagrangean multipliers?
iii. What do the derivatives of $Q^*\left(z_1^T,\ldots z_L^T\right)$ with respect to z_l^T represent?
iv. Consider the simple case of 3 firms, and one input, with technologies given by:

Firm 1: $q_1 \le 20z_1-\frac{\left(z_1\right)^2}{2}$ **; Firm 2:** $q_2 \le 15z_2-\frac{\left(z_2\right)^2}{2}$ **; Firm 3:** $q_3 \le 10z_2-\frac{\left(z_3\right)^2}{2}$

(since there is only input, z_j denotes firm *j's* use of that input).

Derive the industry production function; pay attention to corner solutions.

(b) Consider the case of competitive profit maximizing firms, with the technology of part (a), i.e., $q_j \le F^j\left(z_1^j,\ldots,z_L^j\right)$. All firms face price vectors $\left(p,w_1,\ldots,w_L\right)$, where w_l is the input price.

i. Write the FOC for the profit-maximizing problem for each firm. Are the SOSC satisfied?
ii. Show that the solution to this problem and the solution to the central planner's problem (part a) are identical if $\left(w_l/p\right)=\gamma_l^*$. Does this depend upon the solutions being interior?
iii. Derive the firms' supply curves and the industry supply curve for the technology of part (aiv) above.
iv. What is the relationship between the industry supply curve – derived from profit maximization – and the efficient industry production function $\left(Q^*\right)$ - derived by the central planner? Answer for both the general case and the specific case.

4. ("Exchange Economy") Consider an economy with *L* goods and *H* consumers. Let x_l^h denote consumption of good *l* by household *h*, X_l^T the aggregate availability of good *l*, and $u^h\left(x_1^h,\ldots,x_L^h\right)$ the utility function of individual *h*. A central planner wishes to maximize the utility of person *1*, subject to target utilities for everybody else, and subject to the resource constraints. The maximum problem is:

Max $u^1\left(\vec{x}^1\right)$ s.t. $u^h\left(\vec{x}^h\right)\ge \hat{u}^h$, $h=2,\ldots,H$; $\sum_h x_l^h \le X_l^T$, $l=1,\ldots,L$.

(a) Set up the Lagrangean function for this problem and derive the FOC. Interpret them. (you are deriving the pareto efficient conditions for an exchange economy).

(b) Call the solution x_l^{*h}, λ_l^*, θ^{*h} where the λ_l^* correspond to the resource constraints and the θ^{*h} to the utility constraints (on households). Interpret the meaning of each Lagrangean multiplier.

(c) Suppose you increase the target utility level for person 2 only. In general, do you expect this to change the Lagrangean multipliers and the optimal consumption vectors for all people? Explain.

i. If people had identical and homothetic preferences, how would your answer to (c) be altered?

(d) Consider a given solution, $x_l^{*h}\left(X_1^T,...,X_L^T;\hat{u}^2,...,\hat{u}^H\right)$, with Lagrangean multilpers λ_l^* on the resource constraints. Suppose each household is given income $\omega^h = \sum_l \lambda_l^* x_l^{*h}$ and maximizes utility at prices $p_l = \lambda_l^*$. Compare the solution to the utility maximizing problem (which gives the demand curves) to the optimizing problem solved in part (a).

(e) **Consider the special case of quasi-linear preferences and three goods.** Preferences are: $u^h = x_1^h + \phi^h\left(x_2^h, x_3^h\right)$; where ϕ^h is strictly concave. Aggregate endowments are $\left(X_1^T, X_2^T, X_3^T\right)$

i. Solve the central planner's problem described in part (a) and determine the values of the Lagrangean multipliers. Does the solution depend upon the utility allocation $\left(\hat{u}^h\right)$? {**you may assume all solutions are interior}**.

ii. What is the interpretation of the value of the Lagrangean function at its solution values? {call this $U^*\left(X_1^T, X_2^T, X_3^T;\hat{u}^2,...,\hat{u}^H\right)$ What do the derivatives represent? Be as specific as possible.

iii. Consider a market economy with the same quasi-linear preferences. Suppose each individual has income ω^h, and faces prices $\left(p_1, p_2, p_3\right)$. Find the aggregate demands for good 2 and 3. How does this inverse aggregate demand $\left(p_2\left(X_2^T, X_3^T\right)\right)$ compare to the derivative of U^* from part eii?

(f) Finally, illustrate part (e) for 3 people with the following preferences:
$Person\,1: u^1 = z_1^1 + 2\left(z_2^1 \cdot z_3^1\right)^{1/4}$; $Person\,2: u^2 = z_1^2 + 4\left(z_2^2 \cdot z_3^2\right)^{1/4}$; $Person\,3: u^3 = z_1^3 + \left(z_2^3 \cdot z_3^3\right)^{1/4}$;
(note the superscripts denote the person, not an exponent)

5. **(Deadweight Loss and Second Best)** Consider a simple "general" equilibrium model with three goods. Goods 2 and 3 are produced using good 1; there are J producers, with the same technology, of each good:

$$q_i^j \le 2\left(z_{1,i}^j\right)^{1/2}; \quad j = 1,...,J; \quad i = 2,3$$

In the above equation, q_i^j is firm *j*'s output of good *i*, and $z_{1,i}^j$ is the input of good 1 used by firm *j* producing good *i*. In addition, **there are *J* households**, each with preferences:

$$U^h = x_1^h + 2\left(x_2^h + x_3^h\right) - \left(\frac{\left(x_2^h\right)^2 + \gamma\left(x_2^h\right)\left(x_3^h\right) + \left(x_3^h\right)^2}{4}\right); \quad \gamma \in (-2,2)$$

In the above, x_i^h is household *h's* consumption of good *i*. Each household is endowed with the same amount $\left(m^h = m\right)$ of good *1*, which can be consumed or sold to firms. Firms are competitive, buy input 1 to produce and sell good *i*. Each household has an identical share ownership in each firm, and the profits of the firms are redistributed to households. The following constraints hold:

Budget Constraint: $m + \dfrac{\left(\sum_j \pi_2^j\right) + \left(\sum_j \pi_3^j\right)}{J} + T^h \geq x_1^h + p_2 x_2^h + p_3 x_3^h$

Profit Max: $\pi_i^j = \underset{z_{1,i}^j}{Max}\left\{p_i q_i^j - p_1 z_{1,i}^j\right\} \quad s.t. \quad q_i^j \leq 2\left(z_{1,i}^j\right)^{1/2}; \quad j = 1,...,J; \quad i = 2,3; \quad p_1 \equiv 1$

Resource constraints: Good 1: $\sum_h m^h \geq \sum_h x_1^h + \sum_j z_{1,2}^j + \sum_j z_{1,3}^j$

Goods 2 and 3: $\left(\sum_j q_i^j\right) \geq \left(\sum_h x_i^h\right) \quad i = 2,3$

In the above, we choose good 1 as the numeraire $\left(p_1 \equiv 1\right)$ and T^h is the government transfer to (or taxes from) each household. Initially, assume $T^h = 0 \;\forall h$, and assume there are no other taxes. As usual, households maximize utility and firms maximize profits. **Assume a strict interior solution holds.**

(a) Calculate the demand curves, the supply curves and the equilibrium prices.

(b) Calculate total utility $\left(\sum_h u^h\left(x_1^{h*}, x_2^{h*}, x_3^{h*}\right)\right)$ at this equilibrium {where $\left(x_1^{h*}, x_2^{h*}, x_3^{h*}\right)$ is the consumption vector}. Because of identical quasi-linear preferences total utility can be used as a valid welfare measure.

(c) Suppose a tax, of t_2 per unit, is imposed on good 2; thus, if p_2 denotes the price consumers pay for the good, the net of tax price received by producers is $\left(p_2 - t_2\right)$; equivalently, the profit maximum problem for the firm is $\pi_2^j = \underset{z_{1,2}^j}{Max}\left\{\left(p_2 - t_2\right)q_2^j - p_1 z_{1,2}^j\right\}$. The proceeds of the tax are rebated equally to all consumers, so $T^h = \dfrac{t_2\left(\sum_j q_2^j\right)}{J} \quad \forall h$.

i. Calculate the new equilibrium prices and quantities with this tax.
ii. Using the method from part (b), calculate the loss in welfare due to the tax.
iii. Calculate the deadweight loss from the tax by calculating the changes in producer surplus, consumer surplus and tax revenue in market 2 (using the supply and demand curves). Do you get the same answer as in part (ii)? Does it matter whether, in the utility function, $\gamma \neq 0$?

(d) Finally, assume there is a given tax, t_2, on good 2, and the government is considering a tax or subsidy $\left(t_3\right)$ on good 3 (a subsidy means $t_3 < 0$). Find the equilibrium when both taxes are present.

i. Let $W\left(t_2, t_3\right)$ denote total welfare (or total utility, as defined in part b) as a function of the two taxes. Calculate $\left(\partial W / \partial t_3\right)$, evaluated at $t_3 = 0$ and relate the sign to the sign of γ. Given the tax on good 2, does a tax (or subsidy) to good 3 necessarily lower welfare?

ii. Given the tax on good 2, can you measure the welfare consequences (the deadweight loss) of the tax on good 3 by measuring the changes in consumer surplus, producer surplus and tax revenue in market 3? Explain your answer.