Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2010

Problem Set No. 2
Due by: Thursday, September 2 (4:30 pm)

2.1 Prove that if $\succsim$ is a rational preference relation (complete and transitive) that: (i) $\succ$ is transitive and (ii) $\sim$ is transitive. Are these orderings complete?

2.2 If $\succsim$ is a rational preference relation, prove that $x_1 \succsim x_2, \quad x_2 \succ x_3 \rightarrow x_1 \succ x_3$. If $x_1 \succsim x_2$ and $x_1 \succ x_3$, what can you infer about $x_2 \, ? \, x_3$?

2.3 Consider the properties of local non-satiation, monotone and strongly monotone, as defined in MGW (p.42)

(a) Does local non-satiation imply monotone preferences? Do monotone preferences imply local non-satiation? Prove your answer.

(b) Repeat (a) for the relationship between monotone and strongly monotone preferences.

2.4 Consider the problem: $\underset{x,y}{Max}\ f(x,y) = 2(x+y-3) - (1/2)\left(x^2 + y^2 - xy\right)$ s.t. $x, y \in R_+^2$ and $(x+y) \le A, A > 0$

(a) Must a solution to this problem exist? Will every local maximum be a global maximum?

(b) Find the solution $\left(x^*, y^*\right)$ and show how your answer depends on the value of *A*. Do the necessary conditions of the Kuhn-Tucker condition apply? If so, what is the associated value of λ^* such that $\left(x^*, y^*, \lambda^*\right)$ that solves the K-T conditions? Again, show how your answer depends on *A*.

(c) Suppose *A*=10. Find the solution to the maximum problem assuming the constraint **must** bind – i.e., the domain is $x, y \in R_+^2$ and $(x+y) = 10$. If you solve the equality constrained programming problem, what is the value of λ for this case?

(d) Return to the inequality constrained problem and let *A*=6. Suppose the objective function is unchanged but the domain is now defined by: $x, y \in R_+^2$ and $[6-x-y]^3 \ge 0$. Is this domain any different than that for the original problem with *A=6*? What are the values of x^*, y^* that solve the optimization problem?

(e) Formulate the Lagrangean for the problem of part (d), with constraint $(6-x-y)^3 \ge 0$. Is there any value of λ^* s.t. $\left(x^*, y^*, \lambda^*\right)$ solve the KT conditions? Explain.

(f) Consider the function $w(x,y) = \left(f(x,y)\right)^3$, with $f(x,y)$ as defined above. Is this function $w(x,y)$ concave? Is it quasi-concave? Given the constraint $(x,y) \in R_+^2; (x+y) \le 6$, are there values of (x,y) that maximize $w(x,y)$? Will they be any different than the solution $\left(x^*, y^*\right)$ you found in part (b), with *A=6*?

(g) Formulate the Lagrangean for the problem of part (f), with objective function $w(x,y) = \left(f(x,y)\right)^3$. Are the sufficiency conditions satisfied? Find all solutions to the Kuhn-Tucker conditions. Are they solutions for the original problem? (Hint: are there values of (x,y) s.t. $f(x,y) = 0$ and $x + y \le A$)?

2.5 Consider the problem $\underset{x,y}{Max}\left(x^2+y^2\right)$ s.t. $x,y \in R_+^2$ and $(2x+y)\le 12$. Since the domain is compact and the objective function is continuous, we know there is a global maximum, which is $\left(x^*=0, y^*=12\right)$.

(a) Formulate the Lagrangean function for this problem. Must there be a λ^* such that $\left(0,12,\lambda^*\right)$ solve the K-T conditions? If so, find it.
(b) Use the Lagrangean function in part (a) and find all solutions to the K-T conditions. Do all solutions to these conditions solve the original maximization problem? Explain.

2.6 Cobb-Douglas preferences

Recall the 2-good utility function of the Cobb-Douglas form briefly discussed in class:

$u(\mathbf{x}) = x_1^{\alpha} x_2^{1-\alpha}$, where $\alpha \in (0,1)$.
Let the budget constraint be $(p_1x_1 + p_2x_2)\le w$, $(p_1, p_2) \gg 0, \quad w>0$

(a) Show this utility function is both concave and quasiconcave and derive the demand curves.
(b) Let $v(\mathbf{x}) = \left(x_1^{\alpha} x_2^{1-\alpha}\right)^4$. Is this function concave?; quasi-concave? Does this function exhibit diminishing marginal utility for both goods;? for either good? What are the demands functions for this utility function?

2.7 The CES utility function (MWG, 3.C.6)

Suppose that in a two-commodity world, the consumer's utility function takes the form
$u(x) = \left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{\frac{1}{\rho}} \quad , \quad \alpha_i > 0,\ \alpha_1+\alpha_2 = 1$, and $0 \ne \rho \le 1$
This utility function is known as the *constant elasticity of substitution* (*CES*) utility function.
(a) Show that when $\rho = 1$, indifference curves become linear.
(b) Show that as $\rho \to 0$, this utility function comes to represent the same preferences as the Cobb–Douglas utility function $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.
(c) Show that as $\rho \to -\infty$, indifference curves become "right angles"; that is, this utility function has, in the limit, the indifference map of the Leontief utility function $u(x_1, x_2) = Min\left\{x_1, x_2\right\}$.

2.8 UMP and corner solutions with convex preferences
A consumer maximizes the 2-good utility function:
$u(x_1,x_2) = \left(2\alpha x_1^{1/2} + x_2\right)^2 = 4\alpha^2 x_1 + 4\alpha x_1^{1/2} x_2 + x_2^2;\ \alpha>0, (x_1,x_2)\in R_+^2$.
Prices and income are given p_1, p_2, and w (assume that all are strictly positive).

(a) Formulate the consumer's utility maximization problem and derive the K-T conditions.
(b) Is it possible to have $x_1^* = 0$? If so, derive the required condition(s).
(c) Is it possible to have $x_2^* = 0$? If so, derive the required condition(s).
(d) Solve the K-T conditions to derive the demand curves (pay attention to corner solutions).