Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2011

Problem Set No. 2
Due by: Thursday, September 1 (4:30 pm)

2.1 If $\succsim$ is a rational preference relation, prove that $x_1 \succ x_2, \quad x_2 \succeq x_3 \rightarrow x_1 \succ x_3$. If $x_1 \succsim x_2$ and $x_1 \succ x_3$, what can you infer about x_2 as compared to x_3?

2.2 Consider the properties of local non-satiation, monotone and strongly monotone {MGW (p.42)}
(a) Does local non-satiation imply monotone preferences? Do monotone preferences imply local non-satiation? Prove your answer.
(b) Repeat (a) for the relationship between monotone and strongly monotone preferences.

2.3 Jehle and Reny define convexity (strict convexity) of preferences as follows:

If $x^1 \succeq x^0$, then $tx^1 + (1-t)x^0 \succeq x^0, t \in [0,1]$ {convexity}

If $x^1 \succeq x^0, \; x^1 \neq x^0$, then $tx^1 + (1-t)x^0 \succ x^0, t \in (0,1)$ {strict convexity}

MGW define convexity (strict convexity) of preferences as follows:

If $y \succeq x$, and $z \succeq x$, then $ty + (1-t)z \succeq x, t \in [0,1]$ {convexity}

If $y \succeq x$, and $z \succeq x$, $y \neq z$, then $ty + (1-t)z \succeq x, t \in (0,1)$ {strict convexity}

(a) Assuming preferences are rational, are these definitions equivalent? Prove your answer.
(b) Given an illustration of a utility function that represents preferences which are convex, but not strictly convex.

2.4 Consider the following preferences over consumption bundles in $\Re^2_+$.

$x \succ y \; if \; \left(x_1^{1/2} + x_2^{1/2}\right) > \left(y_1^{1/2} + y_2^{1/2}\right)$

$x \succ y \; if \; \left(x_1^{1/2} + x_2^{1/2}\right) = \left(y_1^{1/2} + y_2^{1/2}\right) \quad AND \quad y_2 > x_2$

{Also, $x \sim x$ }. Note that x_i denotes the amount of good *i* in bundle *x*.

(a) Are these preferences rational (complete, transitive)?
(b) Are they consistent with the monotonicity assumptions? The convexity assumptions?
(c) Draw the upper and lower contour sets to a given bundle x^0.
(d) Can these preferences be represented by a utility function?

2.5 A consumer with preferences represented by differentiable utility function $u(x, y)$ faces the budget constraint $p_x x + p_y y = w$, where *w* is income. For the following situations indicate if the given consumption bundle is a *local maximum* and if it is a *global maximum*. If it is not a local maximum, indicate how the consumption bundle can be altered to increase utility. Also indicate the role convexity plays in your answer. {The notation u_j indicates partial differentiation with respect to good *j*).

(a) $x = (w/p_x), \; y = 0, \quad (u_x/u_y) > (p_x/p_y)$

(b) $x > 0, y > 0; \; (u_x/u_y) < (p_x/p_y)$

(c) $x > 0, y > 0; \; (u_x/u_y) = (p_x/p_y)$

2.6 Consider the problem: $\underset{x,y}{Max}\ f(x,y) = 5(x+y) - 12 - \frac{(x^2 + y^2 - xy)}{2}$ s.t. $x, y \in R_+^2$ and $(x+y) \le A, A > 0$

(a) Must a solution to this problem exist? Will every local maximum be a global maximum?

(b) Find the solution (x^*, y^*) and show how your answer depends on the value of *A*. Do the necessary conditions of the Kuhn-Tucker condition apply? If so, what is the associated value of λ^* such that (x^*, y^*, λ^*) that solves the K-T conditions? Again, show how your answer depends on *A*.

(c) Suppose *A*=30. Find the solution to the maximum problem assuming the constraint **must** bind – i.e., the domain is $x, y \in R_+^2$ and $(x+y) = 30$. If you solve the **equality** constrained programming problem, what is the value of λ for this case?

(d) Return to the inequality constrained problem and let *A*=10. Suppose the objective function is unchanged but the domain is now defined by: $x, y \in R_+^2$ and $[10 - x - y]^3 \ge 0$. Is this domain any different than that for the original problem with *A=10*? What are the values of x^*, y^* that solve the optimization problem?

(e) Formulate the Lagrangean for the problem of part (d), with constraint $(10 - x - y)^3 \ge 0$. Is there any value of λ^* s.t. (x^*, y^*, λ^*) solve the KT conditions? Explain.

(f) Consider the function $w(x,y) = (f(x,y))^3$, with $f(x,y)$ as defined above. Is this function $w(x,y)$ concave? Is it quasi-concave? Given the constraint $(x,y) \in R_+^2; (x+y) \le 10$, are there values of (x,y) that maximize $w(x,y)$? Will they be any different than the solution (x^*, y^*) you found in part (b), with *A=10*?

(g) Formulate the Lagrangean for the problem of part (f), with objective function $w(x,y) = (f(x,y))^3$. Are the sufficiency conditions satisfied? Find all solutions to the Kuhn-Tucker conditions. Are they solutions for the original problem? (Hint: are there values of (x,y) s.t. $f(x,y) = 0$ and $x + y \le 10$)?

2.7 Consider the problem $\underset{x,y}{Max}(x^2 + 2y^2)$ s.t. $x, y \in R_+^2$ and $(x+y) \le 10$. Since the domain is compact and the objective function is continuous, we know there is a global maximum, which is $(x^* = 0, y^* = 10)$.

(a) Formulate the Lagrangean function for this problem. Must there be a λ^* such that $(0, 12, \lambda^*)$ solves the K-T conditions? If so, find it.

(b) Use the Lagrangean function in part (a) and find all solutions to the K-T conditions. Do all solutions to these conditions solve the original maximization problem? Do all represent local maxima? Explain.

2.8 Cobb-Douglas preferences

Recall the 2-good utility function of the Cobb-Douglas form briefly discussed in class:

$u(\mathbf{x}) = x_1^{\alpha} x_2^{1-\alpha}$, where $\alpha \in (0,1)$.

Let the budget constraint be $(p_1 x_1 + p_2 x_2) \le w$, $(p_1, p_2) \gg 0, \quad w > 0$

(a) Show this utility function is both concave and quasiconcave and derive the demand curves.

(b) Let $v(\mathbf{x}) = \left(x_1^{\alpha} x_2^{1-\alpha}\right)^4$. Is this function concave?; quasi-concave? Does this function exhibit diminishing marginal utility for both goods;? for either good? What are the demands functions for this utility function?