Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2011

Problem Set No. 9 Due by: Thursday, November 3, 2011

1. Finish Problems 5-7 from problem set 8.

2. Let $q = f(z_1, z_2) = \left[z_1^2 + z_2^2\right]^{\alpha/2}$; $\alpha > 0$
   (a) Is this function concave?; quasiconcave?
   (b) Is there a solution to the cost minimization problem? If so, find it.
   (c) Is there a solution to the profit maximization problem? If so, find it.

3. {Inferior Inputs – **tedious**} Consider the following production function:
$$q \le \left\{ \frac{(z_1+1) + \left\{(z_1+1)^2 + 4z_2^{1/2}\right\}^{1/2}}{2} - 1 \right\}$$
   (a) Find the first order conditions for the expenditure minimum problem. Show that $z_2 > 0$ for all $q > 0$.
   (b) Show that, for *q* sufficiently large, both inputs will be used.
   (c) Show that when both inputs are used (an interior solution) that z_2 is an inferior input.
   (d) If you can, find the cost curve and try to calculate $\left(\frac{\partial MC}{\partial w_2}\right)$.

4. Let $\hat{x}_i(\vec{w}, Q)$ be the conditional factor demands – i.e., they solve $\underset{\vec{x}}{Min}\left(\sum_i w_i x_i\right) s.t.\quad f(\vec{x}) \ge q$ and let $C(\vec{w}, Q)$ be the resulting cost function for the firm.
   Let $Q^*(p, \vec{w})$ be the firm's profit maximizing supply, i.e., it solves $\underset{Q}{Max}\left\{pQ - C(\vec{w}, Q)\right\}$
   Let $x_i^*(\vec{w}, p)$ be the profit maximizing factor demands, i.e., they solve $\underset{\vec{x}}{Max}\left\{pf(\vec{x}) - \sum_i w_i x_i\right\}$

   (a) What is the relationship be $\hat{x}_i(\vec{w}, Q)$ and $C(\vec{w}, Q)$?
   (b) For the conditional factor demands, does $\left(\partial\hat{x}_i / \partial w_j\right) = \left(\partial\hat{x}_j / \partial w_i\right)$?
   (c) What is the relationship between $\left(\partial Q^* / \partial w_j\right)$ and $\left(\partial\hat{x}_j / \partial Q\right)$?
   (d) What is the relationship between $x_i^*(\vec{w}, p)$ and $\hat{x}_i\left(\vec{w}, Q^*(p, \vec{w})\right)$?
   (e) Use part (d) to show how a factor price change affects the profit-maximizing demand for an input by decomposing this change into the substitution and scale (output) effect, somewhat similar to the substitution and income effects of demand theory. **If an input is inferior, can its demand curve be upward sloping?**

   (f) For profit-maximizing demands, does $\left(\partial x_i^* / \partial w_j\right) = \left(\partial x_j^* / \partial w_i\right)$? Does $sign\left(\partial x_i^* / \partial w_j\right) = sign\left(\partial\hat{x}_i / \partial w_j\right)$? Discuss the implications of this for the possible definitions of substitutes and complements.

5. (Short Run vs. Long Run Cost Functions) Consider a single product firm with the following technology:

$$q = f(z_A, z_B), \quad z_A = \mathbb{R}_+^N, \quad z_B = \mathbb{R}_+^M$$

Let the corresponding factor price vectors be w_A, w_B and define the short run and long run costs as:

$$C^s(q; w_A, w_B, \bar{z}_B) = \underset{z_A}{Min}\left(w_A \cdot z_A + w_B \cdot \bar{z}_B\right) \quad f(z_A, \bar{z}_B) \geq q$$
$$C^L(q; w_A, w_B) = \underset{z_A, z_B}{Min}\left(w_A \cdot z_A + w_B \cdot z_B\right) \quad f(z_A, z_B) \geq q$$

In words, the vector of inputs, z_B is fixed in the short run, but variable in the long run. Let $z_A^L(q; w_A, w_B), z_B^L(q; w_A, w_B)$ denote the long run conditional factor demands, and suppose there exists $(\bar{q}, \bar{w}_A, \bar{w}_B)$ $s.t.$ $z_B^L(\bar{q}, \bar{w}_A, \bar{w}_B) = \bar{z}_B$.

(a) Compare short run and long run costs, and short run and long run conditional factor demands, when evaluated at $(\bar{q}, \bar{w}_A, \bar{w}_B)$. Also, compare the values of short run and long run marginal costs, and the slopes of the marginal cost curves at $(\bar{q}, \bar{w}_A, \bar{w}_B)$.

(b) Let w_A^i denote the price of the i^{th} input in set *A* (so that it is variable in both the short run and long run). Compare how an increase in w_A^1 affects the short run and long run conditional factor demands for inputs in set *A*, and compare the impact of this price increase on short run and long run marginal cost (as always, evaluate this change at $(\bar{q}, \bar{w}_A, \bar{w}_B)$).

(c) Use your result from previous parts to compare the slopes (with respect to output price) of the long run and short run profit maximizing output supply curves, and to compare the slopes (with respect to own input price) of the short run and long run input demand curves. **Do your predictions about the relative values of these slopes hold away from the vector** $(\bar{q}, \bar{w}_A, \bar{w}_B)$

6. There are two firms (*A*,*B*). Each firm uses inputs of x_1 and x_2 to produce output of good *q*. Let $\{x_1^i, x_2^i, q^i\}$ denote firm *i's* input and output vector and assume they have the following technology:

$$q_1^A \leq 10\left(x_1^A \cdot x_2^A\right)^{1/4}; \quad q_1^B \leq 5\left(x_1^B \cdot x_2^B\right)^{1/4}$$

(a) Find each firm's profit maximizing decisions and its maximum profit function, $\pi^i(p, w_1, w_2)$. Also, find the "aggregate" supply and factor demand curves by adding together the supply and demand curves for the two firms.

(b) **Next**, derive the "industry" production function, $q^T(x_1^T, x_2^T)$ by solving the following problem: Given x_1^T, x_2^T, $Max\left(q_1^A + q_1^B\right)$ subject to the resource constraints: $x_i^A + x_i^B \leq x_i^T$, $i = 1, 2$.

(c) **Find the** aggregate supply and input demands $\left(x_1^T, x_2^T, q^T\right)$ that maximize profits for the aggregate production function derived in part (b), and find the maximum (aggregate) profits.

(d) **Show** that the aggregate maximum profit function, and the corresponding output supply and input demands, are just the sum of the individual firm's rules (e.g., $\pi^T = \pi^A + \pi^B$).

(e) An aggregate netput vector y^T is said to be efficient if there does not exist a technologically feasible vector y' (in the aggregate production set) such that $y' \geq y^T, \quad y' \neq y^T$ {in this case, the netput vector is $y = \{-x_1, -x_2, q\}$ }. Use the previous parts to argue that individual profit maximization leads to an aggregate production vector that is efficient.

7. (Generalization of Problem 6 above): There are *J* firms, each of which produces the same good (*q*) using the same set of inputs $\left(z_1, ..., z_L\right)$. The production set for each firm is strictly concave and given by:

$$q_j \leq F^j\left(z_1^j, ..., z_L^j\right)$$

(a) A central planner, with given amounts of each input, wishes to maximize output of good *q;* the optimization problem is written as:

$$L = \sum_j q_j + \sum_l \gamma_l \left(z_l^T - \sum_j z_l^j\right) = \sum_j F^j\left(z_1^j, ..., z_L^j\right) + \sum_l \gamma_l \left(z_l^T - \sum_j z_l^j\right)$$

where z_l^T is the availability of each input. The Lagrangean, evaluated at the solution, gives the industry production function: $Q^*\left(z_1^T, ...z_L^T\right) = L\left(z_1^T, ..z_L^T; z_l^{*j}, \gamma_l^*\right)$; $\left(z_l^{*j}, \gamma_l^*\right)$ is the solution vector.

i. Write the FOC for the optimizing problem and interpret. Are the SOSC satisfied?
ii. What is the meaning of the Lagrangean multipliers?
iii. What do the derivatives of $Q^*\left(z_1^T, ...z_L^T\right)$ with respect to z_l^T represent?
iv. Consider the simple case of 3 firms, and one input, with technologies given by:

Firm 1: $q_1 \leq 20z_1 - \frac{(z_1)^2}{2}$ **; Firm 2:** $q_2 \leq 15z_2 - \frac{(z_2)^2}{2}$ **; Firm 3:** $q_3 \leq 10z_2 - \frac{(z_3)^2}{2}$

(since there is only input, z_j denotes firm *j's* use of that input).

Derive the industry production function; pay attention to corner solutions.

(b) Consider the case of competitive profit maximizing firms, with the technology of part (a), i.e., $q_j \leq F^j\left(z_1^j, ..., z_L^j\right)$. All firms face price vectors $\left(p, w_1, ..., w_L\right)$, where w_l is the input price.

i. Write the FOC for the profit-maximizing problem for each firm. Are the SOSC satisfied?
ii. Show that the solution to this problem and the solution to the central planner's problem (part a) are identical if $\left(w_l / p\right) = \gamma_l^*$. Does this depend upon the solutions being interior?
iii. Derive the firms' supply curves and the industry supply curve for the technology of part (aiv) above.
iv. What is the relationship between the industry supply curve – derived from profit maximization – and the efficient industry production function $\left(Q^*\right)$ - derived by the central planner? Answer for both the general case and the specific case.