

Problem Set No. 10 Due by: Thursday, November 10

1. Do Questions 6 & 7, Problem Set 9
2. Let $c(w, q)$ denote the cost function of a competitive firm, where q is output and w is the vector of input prices. Assume that it takes the following form:

$$c(w, q) = \begin{cases} 4w_1 + q^2 \sqrt{w_1 w_2} & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

- (a) Find the firm's profit-maximizing supply function. Now suppose that $w_1 = w_2 = 1$, and let p represent output price. What is the firm's optimal output if $p = 6$? What about when $p = 3$?
 - (b) Assume that there is free entry in this competitive market, and that input prices are $w_1 = w_2 = 1$. What is the long-run supply correspondence for this industry?
 - (c) The output of this industry is demanded by 1,000 consumers, each with indirect utility function $V_i = \omega_i - p + p^2/10$, where i indexes consumers and ω denotes income (measured in units of a numeraire good). Input prices are still assumed fixed at $w_1 = w_2 = 1$. Determine the long-run equilibrium (including the long-run number of firms) in this market.
3. Consider a competitive industry in long run equilibrium. All firms are identical, each with cost function $C(w, q)$, where q denotes the output of one firm, and w is the vector of input prices. This cost function displays a U-shaped average cost and a strictly increasing marginal cost. The (downward sloping) market demand for this industry is written as $x(p, \alpha)$, where p denotes the price for the industry output and α is a shift parameter.

All input prices **except** w_1 **are exogenous**. However, this industry is the only user of input 1, and the market supply of this input is given by $S(w_1)$, where $S'(w_1) > 0$.

The industry long-run equilibrium is characterized by the values of $\{p^*, q^*, J^*, w_1^*\}$, where J denotes the number of firms. [Strictly speaking J is an integer, but you can ignore that and treat J as a real number].

- (a) Write down the system of equations that define the long-run equilibrium. Briefly discuss the rationale behind each of the equations. Also, show how an increase in demand (an increase in α , since $(\partial x / \partial \alpha) > 0$) affects the equilibrium.
- (b) Now assume w_1 is exogenous (i.e., $S(w_1)$ is infinitely elastic). Use comparative statics on the system of equations derived in (a) – with w_1 constant – to determine the impact on the long run equilibrium of an increase in an input price w_k , assume input k is an inferior input [Recall that an

input is said to be inferior if the cost-minimizing input demand is negatively related to output, i.e., $\partial x_k^c(w, q) / \partial q \leq 0$. Specifically, determine the signs of $\partial p^* / \partial w_k$, $\partial q^* / \partial w_k$ and $\partial J^* / \partial w_k$.

4. Consider a simple model with two goods (x_1, x_2) . There are H firms, each using good 1 to produce good. There are also H identical consumers, each with endowment of goods $(e_1, e_2) = (10, 0)$; in addition, each consumer owns an each share $(1/H)$ of each firm, so any profits of the firm are redistributed to consumers. Consumer preferences, cost functions (technology) and resource constraints are given by:

$$\text{Consumer Preferences: } U^h = c_1^h + A \ln c_2^h \quad h = 1, \dots, H;$$

$$\text{Firms producing good 1: } c^j(q_2^j) = p_1 \left(\frac{(q_2^j)^2}{2} \right); \quad j = 1, \dots, H$$

$$\text{Resource constraints: } \sum_j q_2^j \geq \sum_i c_2^i, \quad i = 1, 2; \quad 10H - \sum_h c_1^h - \sum_j x_1^j \geq 0$$

Note that $x_1^j = \left(\frac{(q_2^j)^2}{2} \right)$ is the amount of good 1 that firm j uses as an input to produce good 2.

- a) Write the budget constraint for the household and assuming utility maximization by households and profit maximization by firms, find the equilibrium.

- b) Suppose the government imposes a percent tax on good 2 bought by households so that if p_2 is the price received by firms, the price paid by households is $p_2(1 + \tau)$. All of the tax revenue is rebated on an equal per capita basis to consumers that is (essentially) independent of their own purchases so

$$\text{the tax rebate to each household is } T^h = \left(\frac{\sum_{k=1}^H \tau p_2 c_2^k}{H} \right). \text{ Calculate the new equilibrium as a function}$$

of the tax rate.

- c) Calculate the change in consumer utility due to the tax. Relate this to the deadweight loss you would calculate using supply and demand curves.

5. **(Deadweight Loss and Second Best)** Consider a simple “general” equilibrium model with three goods. Goods 2 and 3 are produced using good 1; there are J producers, with the same technology, of each good:

$$q_i^j \leq 2(z_{1,i}^j)^{1/2}; \quad j = 1, \dots, J; \quad i = 2, 3$$

In the above equation, q_i^j is firm j 's output of good i , and $z_{1,i}^j$ is the input of good 1 used by firm j producing good i . In addition, **there are J households**, each with preferences:

$$U^h = x_1^h + 2(x_2^h + x_3^h) - \left(\frac{(x_2^h)^2 + \gamma(x_2^h)(x_3^h) + (x_3^h)^2}{4} \right); \quad \gamma \in (-2, 2)$$

In the above, x_i^h is household h 's consumption of good i . Each household is endowed with the same amount $(m^h = m)$ of good 1, which can be consumed or sold to firms. Firms are competitive, buy input 1

to produce and sell good i . Each household has an identical share ownership in each firm, and the profits of the firms are redistributed to households. The following constraints hold:

$$\text{Budget Constraint: } m + \frac{\left(\sum_j \pi_2^j\right) + \left(\sum_j \pi_3^j\right)}{J} + T^h \geq x_1^h + p_2 x_2^h + p_3 x_3^h$$

$$\text{Profit Max: } \pi_i^j = \text{Max}_{z_{1,i}^j} \left\{ p_i q_i^j - p_1 z_{1,i}^j \right\} \quad \text{s.t. } q_i^j \leq 2 \left(z_{1,i}^j \right)^{1/2}; \quad j=1, \dots, J; \quad i=2,3; \quad p_1 \equiv 1$$

$$\text{Resource constraints: Good 1: } \sum_h m^h \geq \sum_h x_1^h + \sum_j z_{1,2}^j + \sum_j z_{1,3}^j$$

$$\text{Goods 2 and 3: } \left(\sum_j q_i^j \right) \geq \left(\sum_h x_i^h \right) \quad i=2,3$$

In the above, we choose good 1 as the numeraire ($p_1 \equiv 1$) and T^h is the government transfer to (or taxes from) each household. Initially, assume $T^h = 0 \forall h$, and assume there are no other taxes. As usual, households maximize utility and firms maximize profits. **Assume a strict interior solution holds.**

(a) Calculate the demand curves, the supply curves and the equilibrium prices.

(b) Calculate total utility $\left(\sum_h u^h \left(x_1^{h*}, x_2^{h*}, x_3^{h*} \right) \right)$ at this equilibrium {where $\left(x_1^{h*}, x_2^{h*}, x_3^{h*} \right)$ is the consumption vector}. Because of identical quasi-linear preferences total utility can be used as a valid welfare measure.

(c) Suppose a tax, of t_2 per unit, is imposed on good 2; thus, if p_2 denotes the price consumers pay for the good, the net of tax price received by producers is $(p_2 - t_2)$; equivalently, the profit maximum problem for the firm is $\pi_2^j = \text{Max}_{z_{1,2}^j} \left\{ (p_2 - t_2) q_2^j - p_1 z_{1,2}^j \right\}$. The proceeds of the tax are rebated equally

$$\text{to all consumers, so } T^h = \frac{t_2 \left(\sum_j q_2^j \right)}{J} \quad \forall h.$$

- i. Calculate the new equilibrium prices and quantities with this tax.
- ii. Using the method from part (b), calculate the loss in welfare due to the tax.
- iii. Calculate the deadweight loss from the tax by calculating the changes in producer surplus, consumer surplus and tax revenue in market 2 (using the supply and demand curves). Do you get the same answer as in part (ii)? Does it matter whether, in the utility function, $\gamma \neq 0$?

(d) Finally, assume there is a given tax, t_2 , on good 2, and the government is considering a tax or subsidy (t_3) on good 3 (a subsidy means $t_3 < 0$). Find the equilibrium when both taxes are present.

- i. Let $W(t_2, t_3)$ denote total welfare (or total utility, as defined in part b) as a function of the two taxes. Calculate $(\partial W / \partial t_3)$, evaluated at $t_3 = 0$ and relate the sign to the sign of γ . Given the tax on good 2, does a tax (or subsidy) to good 3 necessarily lower welfare?
- ii. Given the tax on good 2, can you measure the welfare consequences (the deadweight loss) of the tax on good 3 by measuring the changes in consumer surplus, producer surplus and tax revenue in market 3? Explain your answer.