

Final exam  
Answer Any Three Questions

1. Consider a monopoly electric company that sells electricity to two types of customers: “rich” and “poor” customers. The demand functions for these customers (which are derived from quasilinear preferences for each consumer) are:

$$x_p = 40 - p_p$$

$$x_r = 60 - p_r$$

where  $p_p, p_r$  is the price (per unit) paid by poor and rich customers, respectively, and  $x_p, x_r$  are the quantities (of electric power) demanded by each type of customer. The firm’s cost function is:

$$C(x) = 2x; \quad x = (x_r + x_p)$$

- (a) Assume the firm must charge the same price to all consumers and that there are no taxes or subsidies ( $p_p = p_r = p$ ). Solve for the firm’s profit maximizing output and price. **(7 points)**
- i. Given these demands (preferences) and costs, find the efficient output level – i.e., the output level that maximizes the sums of consumer surpluses and producer surplus (profit). Calculate the deadweight loss due to the monopoly behavior of the firm. **(6 points)**
- (b) Continue to assume the monopoly electric company must charge all consumers the same price ( $p$ ), but also assume the government subsidizes, at rate  $s$  per unit, the electric consumption of poor households. Thus, the net price paid by poor households is  $(p - s)$ , while rich households pay  $p$ . Find the profit-maximizing price and output for the firm. **(7 points)**
- i. How does the subsidy to poor consumers affect (i)total output, (ii)consumption of poor households and (iii)consumption of rich households? Will a small subsidy necessarily improve “efficiency” (defined as the sum of rich and poor consumer surpluses plus firm profits less the tax cost of the subsidy)? Only an intuitive answer is expected. **(6 points)**
- ii. Suppose the firm can charge rich and poor consumers different prices (and resale is not feasible). How would the government subsidy to poor consumers affect (i)total output and (ii)consumption of each type of household? Will a small subsidy necessarily improve efficiency? Only an intuitive answer is expected. **(7 points)**

2. Consider a competitive industry in long run equilibrium. All firms are identical with the following cost function:

$$C(w_1, w_2, q) = \begin{cases} F + \frac{q^2 (w_1 \cdot w_2)^{1/2}}{2} & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

where  $q$  denotes the firm's output,  $w_1, w_2$  the prices of the two inputs used to produce  $q$ , and  $F > 0$  the fixed cost associated with setting up production. Let  $D(p, \alpha)$  denote the demand for the industry's output, where  $p$  is price and  $\alpha$  a demand shift parameter. The industry long-run equilibrium is characterized by  $(p^*, q^*, J)$  where  $p^*$  is equilibrium price,  $q^*$  is equilibrium output and  $J$  is the number of (identical) firms (you may treat  $J$  as a continuous variable).

- (a) Assuming  $(w_1, w_2)$  are exogenous, derive the long run industry supply curve, and show how the long run equilibrium triple  $(p^*, q^*, J)$  are determined. **(8 points)**

- i. Compare the effects of an increase in demand (an increase in  $\alpha$  where  $D_\alpha > 0$ ) on:  
 (1) output per firm, (2) firm's profits; (3) equilibrium price; and (4) total industry output in the short run, when the number of firms is fixed, and in the long run. **(9 points)**

- (b) Assume that  $w_1 = 1$  is exogenous, but that  $w_2$  (the price of input two,  $z_2$ ) is affected by this industry's demand for that input. Input 2 ( $z_2$ ) is produced by a competitive profit-maximizing industry with the following (long run) supply curve:

$$z_2^S = S(w_2) = Aw_2; \quad A > 0$$

Recalling that one can get the firm's conditional input demand from the cost function, **derive** the long run industry supply curve for good  $q$ . (Remember this requires demand equal supply in the market for input 2. If you cannot derive the long run industry supply curve, write down the set of equations which would allow you to derive it). **(8 points)**

- i. What does the area next to the long run industry supply curve for good  $q$  measure? Be specific and, if possible, prove your answer. **(8 points)**

3. An individual with wealth  $W_0$  faces a possible loss,  $L$  (due to fire) with exogenous probability  $\pi$ . The individual can reduce the magnitude of the loss by spending money ( $m$ ) on fire retardation devices, such as water sprinklers, fire doors, etc. Thus, the person's realized wealth is:

$$W = \begin{cases} W_0 - L(m) - m & \text{with probability } \pi; \quad L = L_0 e^{-\alpha m}; \quad \alpha > 0 \\ W_0 - m & \text{with probability } (1-\pi) \end{cases}$$

- (a) Let the person's Bernoulli utility function be  $u(W)$ , with  $u' > 0 > u''$ . Assuming the person chooses spending on  $m$  to maximize expected utility, set up the optimization problem and derive the first order conditions. **(8 points)**
- i. Compare the amount the risk averse person invests in reducing loss (*i.e.*, the optimal value of  $m$ ) to the amount that would be invested by a risk neutral person ( $u'' = 0$ ). **(8 points)**
- ii. How will an increase in  $W_0$  affect the risk averse person's optimal value of  $m$ ? Be precise and **prove** your answer. **(8 points)**
- (b) Assume the risk averse individual can also buy insurance against fire loss. Let  $B$  denote the amount of insurance she buys, and  $q$  be the cost per dollar of insurance. Realized wealth is:

$$W = \begin{cases} [W_0 - L(m) - m + B - qB] & \text{with probability } \pi; \quad L = L_0 e^{-\alpha m}; \quad \alpha > 0 \\ [W_0 - m - qB] & \text{with probability } (1-\pi) \end{cases}$$

- Assuming the insurance is actuarially fair** ( $q = \pi$ ), find the values of  $(m, B)$  that maximize expected utility. **(9 points)**

4. Answer all parts.

(a) Consider a competitive firm producing *shirts*. The firm has cost function  $C(q, \bar{w})$ :

$$C(q, \bar{w}) = \begin{cases} 0 & \text{if } q = 0 \\ F + \phi(q, \bar{w}) & \text{for } q > 0 \end{cases}$$

Here,  $q$  is the firm's output of shirts,  $F > 0$  is the fixed cost required to set up production and  $\bar{w} \in \mathfrak{R}_{++}^n$  is the exogenous factor price vector. Assume input 2 is an **inferior input** in some domain of  $(q, \bar{w})$  - in this domain the cost-minimizing demand for input 2 decreases as output increases. Suppose that the shirt industry is composed of identical firms, and industry equilibrium output and price are determined by setting industry supply equal to the demand for shirts,  $D(p)$ .

- i. In the short run, when the number of firms is fixed, how will an exogenous increase in  $w_2$  affect the equilibrium output and price of shirts (assuming  $w_2$  is in the domain where input 2 is an inferior input)? Prove your answer. **(9 points)**
- ii. In the long run, when there is free entry and exit of firms and profits must be zero, how does the increase in  $w_2$  affect the equilibrium output and price of shirts (assuming input 2 is an inferior input)? Prove your answer. **(8 points)**

(b) Suppose a firm uses  $n$  inputs  $(z_1, \dots, z_n)$  to produce a good  $q$ . The firm's production function  $q \leq f(z_1, \dots, z_n)$  is strictly quasi-concave, increasing in its arguments and *homothetic*.

- i. Set up the cost minimization problem for the firm. What are the properties of the cost function and of the conditional input demands? Be as specific as possible. **(9 points)**
- ii. If the production function is strictly concave - and not just strictly quasi-concave - what additional restriction does this place on the cost function? Be specific. **(7 points)**

5. An individual with income  $w$  has utility function:  $u(\bar{x}) = 2^{3/2} (x_2 \cdot x_3)^{1/4} + x_1$ . Prices are given by  $\bar{p} \equiv (p_1, p_2, p_3)$ , with  $\bar{p} \in \mathfrak{R}_{++}^3$ . {You may assume throughout that  $w$  is sufficiently large so that an interior solution, in which all three goods are consumed, occurs}.
- (a) Derive the compensated demands for goods 2 and 3. Using your results, what is the sign pattern of the substitution matrix? Be as specific as possible. **(10 points)**
- (b) Suppose initially  $w = 40$ , and  $(p_1, p_2, p_3) = (4, 4, 4)$ . Assume that, by joining a club, the consumer can shop at prices  $(p_1, p_2, p_3) = (4, 1, 1)$ . Using the compensated demands derived in part (a), calculate the maximum fee the person would pay to join the club. **(9 points)**
- (c) Assume the person currently lives in Ames, has income  $w = 40$  and faces prices  $(p_1, p_2, p_3) = (4, 4, 4)$ . The person has the opportunity to move to New York, where prices are higher but her income will only become known after she moves there. In particular, prices in New York are  $(p_1, p_2, p_3) = (8, 8, 8)$ , whereas her income in New York will be, with equal probability, either  $w = 140$  or  $w = 40$ . Assuming she is an expected utility maximizer, should she make the move, assuming she will not be able to return to Ames if her New York income is low? **(7 points)**
- (d) Finally, suppose the person's preferences were  $v(x) = \ln[u(x)] = \ln[2^{3/2} (x_2 \cdot x_3)^{1/4} + x_1]$ . Would this transformation of preferences affect your answers to the previous parts of this question? {You do not need a numerical answer – just a descriptive answer}. **(7 points)**