Economics 601, Microeconomic Analysis I — Harvey Lapan
Fall 2009

Midterm 2
Answer Any Three Questions

1. Answer all parts.

(a) Consider a consumer whose utility depends on two goods, $\{x_1, x_2\}$. Her Hicksian demand for good 1 is given by: $x_1^h = U\left(\frac{p_2}{p_1}\right)^{1/2}$, where $U$ is utility, and $p_i$ is the price of good $i$. Currently, the individual has income $w = 72$, prices are $(p_1, p_2) = (9,4)$ and the person's optimal consumption bundle is $(x_1, x_2) = (4,9)$.

i. A new discount store opens that allows her to buy $x_1$ at a price of $p_1 = 4$ per unit. What is the maximum amount she would pay for the opportunity to shop at this store? **(7 points)**

ii. Suppose, instead of offering good 1 at $p_1 = 4$, the new store has a more complex offer: you pay a price of **12 each** for the first **4** units of good 1 bought there, while additional units purchased in excess of **4** cost only **1** each. Thus, if you buy $\hat{x}_1$ units of good 1 at that store, the **total cost** of your purchases of good 1 there is: $\left\{12 \cdot Min\left[\hat{x}_1, 4\right] + 1 \cdot Max\left[\left(\hat{x}_1 - 4\right), 0\right]\right\}$. Your income and the price of good 2 remain the same $\left(w = 72, p_2 = 4\right)$. {The customer **cannot** resell purchases of good one to someone else – the good is not transferable}.

iia) Draw the budget set for this problem. Is it convex? Is there a unique local optimum to the utility maximization problem? {you do not need to find the optimum}. **(4 points)**

iib) Will the person choose to shop at the new store (her alternative is to buy as much as she wants of good 1 at a price $p_1 = 9$)? Justify your answer. **(7 points)**

(b) A consumer has preferences given by: $U = \left(x_1^{-1} + x_2^{-1}\right)^{-1}$ (this is a quasi-concave function). Prices are given by $(p_1, p_2)$ and the person's income is derived from endowments $(e_1, e_2)$.

i. Are these preferences homothetic? For the Marshallian demands, will $\left(\frac{\partial x_1}{\partial p_2}\right) = \left(\frac{\partial x_2}{\partial p_1}\right)$, given that income is derived from endowments? **(6 points)**

ii. Assume $e_1 = 3e_2$. Find the Marshallian demand for $x_1$ and calculate the slope $\left(\frac{\partial x_1}{\partial p_1}\right)$. Is the slope negative everywhere? If you answer no, find the domain (of prices) for which the demand curve is positively sloped. **(9 points)**

2. A competitive profit-maximizing firm produces a good, $q$, using a monotonic, concave two input production function $f(z_1, z_2)$. Let $p$ denote output prices, and $(w_1, w_2)$ denote input prices. Let $\pi(p, w_1, w_2)$ denote the firm's (maximum) profit function.

(a) An economist postulates the profit function has the form: $\pi(p, w_1, w_2) = 2p^2\left[w_1^a + w_2^a\right]^{(\mu/a)}$. What restrictions must hold on $\mu$ and $a$ for this profit function to be consistent with theory? Explain. **(7 points)**

(b) Let $\pi(p, w_1, w_2) = 2p^2\left[w_1^{(1/2)} + w_2^{(1/2)}\right]^{-2}$. Derive: (1)the cost function that is dual to $f(z_1, z_2)$; (2)the conditional input demand functions; and (3)the dual production function. **(18 points)**

(c) [Use the profit function (and dual technology) from part (b)]. Let inputs prices be $w_1 = w_2 = 1$ and output price $p = 10$. Walmart offers to pay the firm a price of $p = 20$ BUT the firm must supply Walmart with 50 units at that price (and must produce the good itself). Calculate whether the firm should accept Walmart's offer. **(8 points)**

3. Consider a firm that has the following two input production function: $q \le f(z_1, z_2) \equiv \left(4z_1^2 + z_2^2\right)^{\lambda/2}$.

(a) Calculate the (local) returns to scale for this production function. Under what condition does it satisfy non-increasing returns to scale? **(7 points)**

(b) Let $\lambda = (1/2)$ in the production function above. **(1)**Does the production function exhibit non-increasing returns to scale? **(2)**Is the production function concave? That is, is the production set $Y = \left\{(-z_1, -z_2, q) \in R^3 \ s.t.\ z_1 \ge 0,\ z_2 \ge 0,\ and\ q \le \left(4z_1^2 + z_2^2\right)^{1/4}\right\}$ a convex set? Carefully justify your answer. **(8 points)**

(c) Derive the cost function for this production function (with $\lambda = (1/2)$). Is there a profit maximizing solution for the competitive firm? Explain. **(10 points)**

(d) Let $\lambda = 2$ and $g(z_1, z_2) = h(f(z_1, z_2)) = \dfrac{Af(z_1, z_2)}{1 + f(z_1, z_2)} = \dfrac{A\left(4z_1^2 + z_2^2\right)}{1 + \left(4z_1^2 + z_2^2\right)}$, $A > 0$. {Note that $h(\cdot)$ is a positive monotonic transformation of the production function}. What is the cost function dual to $g(z_1, z_2)$ and what does the average cost curve look like? What is the value of minimum average cost? Be specific. {for simplicity, let $w_1 = w_2 = 1$}. **(8 points)**

4. Answer All Parts

(a) A competitive, profit maximizing firm produces an output, $q$, using two inputs $(z_1, z_2)$. Output and input prices are given by $(p, w_1, w_2)$. The firm's profit-maximizing output supply and input demands are given by $q^*(p, w_1, w_2)$, $z_i^*(p, w_1, w_2)$, $i = 1, 2$. At prices $(p, w_1, w_2) = (2,1,1)$ we are given that $(\partial q^* / \partial p) = 4$, $(\partial z_1^* / \partial w_1) = -1$

i. Given the above information, what is the feasible range of values for $(\partial q^* / \partial w_1)$? **(5 points)**

ii. Suppose we also know that at $(p, w_1, w_2) = (2,1,1)$, $(\partial z_2^* / \partial w_1) = 3$. Find $(\partial q^* / \partial w_1)$ at that price vector. **(8 points)**

(b) Consider a competitive profit-maximizing firm that produces an output, $q$, using three inputs, $(z_1, z_2, z_3)$. Output and input prices are given by $(p, w_1, w_2, w_3)$ and the firm's strictly concave production function by $f(z_1, z_2, z_3)$. In the short run, $z_3$ is fixed (at $\overline{z}_3$), while $(z_1, z_2)$ are variable inputs whereas in the long run all inputs are variable. Let $\pi^s(p, w_1, w_2, w_3, \overline{z}_3)$ denote the short run (restricted) profit function, and $\pi^L(p, w_1, w_2, w_3)$ the long run profit function. Similarly, let $(\hat{q}^s, \hat{z}_1^s, \hat{z}_2^s)$ denote the short run profit-maximizing choices of the firm and $(\hat{q}^L, \hat{z}_1^L, \hat{z}_2^L, \hat{z}_3^L)$ the long run profit maximizing choices.

i. What can you conclude about the sign of $\left[(\partial \hat{z}_1^L / \partial w_1) - (\partial \hat{z}_1^s / \partial w_1)\right]$ and what can you conclude about the sign of $\left[(\partial \hat{q}^L / \partial w_1) - (\partial \hat{q}^s / \partial w_1)\right]$? Be as specific as possible. **(6 points)**

ii. Suppose the production function is given by $f(z_1, z_2, z_3) = h(z_1, z_2) + g(z_2, z_3)$, where $h(\cdot)$ and $g(\cdot)$ are both monotonic increasing and strictly concave functions and $(\partial^2 g / \partial z_2 \partial z_3) < 0$. For this case what can you conclude about the sign of $\left[(\partial \hat{z}_1^L / \partial w_2) - (\partial \hat{z}_2^s / \partial w_2)\right]$? Be as specific as possible. **(7 points)**

iii. Suppose the short run restricted profit function is: $\pi^s = \dfrac{2p^2}{w_1} + \dfrac{4p^{4/3}\overline{z}_3^{1/3}}{w_2^{1/3}} - w_3\overline{z}_3$.

Compare the slopes of the short run and long run supply curves $\left[(\partial \hat{q}^L / \partial p) - (\partial \hat{q}^s / \partial p)\right]$ and also determine the sign of $\left[(\partial \hat{z}_1^L / \partial p) - (\partial \hat{z}_1^s / \partial p)\right]$. **(7 points)**