Economics 601 Harvey Lapan
Microeconomic Analysis I Fall 2009

Problem Set No. 12: Due by: Friday, December 11, 2009

1. [INSURANCE] Individual *A*, with Bernoulli utility function $u^A(w)$ has initial wealth w_0. With probability π *A* will face a loss *L*, while with probability $(1-\pi)$ no loss will occur. A can buy insurance that will pay *I* if the loss occurs, and will pay nothing otherwise. The insurance costs q per dollar of insurance purchased, so the cost of the policy is qI. Thus, ex post wealth is as follows:

If a loss occurs: $w_L = w_0 - L + I(1-q)$; If no loss: $w_N = w_0 - qL$

A's problem is to choose *I* to maximize expected utility.

- **(a)** Set up the optimization problem and derive the FOC. Are the SOC satisfied?
- **(b)** Under what condition will A buy full insurance $(I = L)$?
- **(c)** Suppose $q > \pi$. Under what condition will A buy some insurance?
- **(d)** Suppose $q > \pi$ and that *A* buys some insurance. Suppose *A* has a friend, *B*, who has the same wealth, faces the same risk of loss *L*, and can buy insurance under the same conditions. *B's* utility function is $u^B(w) = \left(u^A(w)\right)^{1/2}$. What can you conclude about *B*'s purchase of insurance, as compared to *A*? Be as specific as possible.
- **(e)** Suppose *A* has a third friend, *C*, who has the same income, faces the same loss and can buy from the same insurance policy. *C's* Bernoulli utility function is: $u^c(w) = \alpha + \beta u^A(w); \quad \beta > 0$. How will *C*'s insurance decision compare to *A*'s decision?
- **(f)** Finally, suppose $u^A(w) = \ln(w)$. Find *A*'s optimal insurance decision, assuming $q > \pi$. Show how I changes as *q* increases; as w_0 increases. What happens as *L* increases? (Is $\left(\partial I^* / \partial L\right) < 1$?)

2. An individual who lives two periods has the utility function $U(c_1, c_2) = \phi(c_1) + \beta\phi(c_2)$, where c_i is consumption of a single composite good in period *i* (*i*=1,2). The person's income in each period is y_i, the price of the composite good in each period *i* is normalized to one, and the one period interest rate is *r*. The person always knows y_1 when choosing c_1 but y_2 and r may not be known. The intertemporal budget constraint is: $(y_1 - c_1)(1+r) + (y_2 - c_2) \geq 0$

- **(a)** Assume that when the individual chooses c_1, he knows r and y_2 with certainty. Write the FOC for optimal period one consumption.
- **(b)** Next, assume that when he chooses c_1 he knows *r* but does **not** know y_2 with certainty. Assuming $E(y_2)$ is the same as in part (a) – when there was no uncertainty – how does the income uncertainty affect period 1 consumption?
- **(c)** Assume y_2 is known with certainty when c_1 is chosen, but that *r*, the (real) return on savings, is not known. Assuming $E(r)$ is the same as in part (a) – when there was no uncertainty – can you tell how the interest rate uncertainty affects period 1 consumption? Be specific.
- **(d)** Assume $U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$. Answer parts (a)-(c) for this specific function. For the case of interest rate uncertainty, relate your answer to whether the person borrows or saves in period 1 (i.e., to whether $c_1 > y_1$ or $c_1 < y_1$

3. An individual *A* owns an assets *x* that yields a risky return. Let $F(x)$ denote the distribution function and let $\bar{x}$ denote the expected return. The certainty equivalent to this risky asset is the amount of money that would make *A* indifferent between the risky asset and the certain amount:

$$c(F,u): \quad u(c(F,u)) = \int u(x)\,dF(x)$$

- **(a)** Compare $c(F,u)$ to $\bar{x} \equiv \int x\,dF(x)$. Relate your answer to the curvature of *u*.
- **(b)** Let $x = \bar{x} + \alpha\varepsilon; \quad E(\varepsilon) = 0, \quad \alpha > 0$. $\bar{x}$ is the expected return of the risky asset, ε is a mean zero random variable and α is a scalar. Increases in α represent a mean-preserving spread of the distribution (an increase in "risk"). Show how increases in α affect the certainty equivalent.
- **(c)** Show how the certainty equivalent changes with $\bar{x}$. In particular, determine if $(dc/d\bar{x}) > 1$.
- **(d)** Suppose that *A's* friend, *B*, has the Bernoulli utility function $w(x) = \phi(u(x)); \quad \phi' > 0$. How will *B's* certainty equivalent for this asset compare to that of *A*? Be specific.

4. An individual with initial wealth w_0 can invest in a safe asset that yields gross return $r^s \geq 1$, and in a risky asset that yields random gross return r. Let a denote the amount invested in the risky asset, so $(w_0 - a)$ is invested in the safe asset. The individual chooses *a* to maximize the expected utility of ex post wealth, $\tilde{w} = ra + r^s(w_0 - a) = r^s w_0 + (r - r^s)a$. The person's Bernoulli utility function is $u(x)$ so the problem is $\underset{a}{Max}\,\underset{r}{E}(u(\tilde{w})) = \underset{a}{Max}\int_r u\left(r^s w_0 + (r - r^s)a\right)dF(r)$

- **(a)** Write the FOC for this problem and confirm the SOC. If $E(r) > r^s$ will the person invest all his money in the risky asset? What happens if $E(r) \leq r^s$?
- **(b)** Let $a^*(w_0, F, u)$ be the optimal decision for the person with utility function *u*, and wealth w_0. Suppose another individual with the same wealth and facing the same asset choice has Bernoulli utility function $\lambda(x) = \phi(u(x)), \quad \phi' > 0 > \phi''$. How will this person's optimal portfolio compare to the person with utility function *u*?
- **(c)** Return to person with utility function *u*. Show how an increase in w_0 affects his portfolio decision – i.e., determine the signs of $\left(\partial a^*/\partial w_0\right)$ and $\left(\partial\left[a^*/w_0\right]/\partial w_0\right)$. Relate your answer to the concepts of Absolute Risk Aversion and Relative Risk Aversion.

5. Consider a potential firm deciding whether to enter the market for a good *before* the price of that good is known (think of demand uncertainty). Decisions are made in the following sequence: In stage 1, the firm decides whether to enter the market or not; if the firm enters, it has a sunk cost of *F*; in stage 2, the firm discovers what output price *(p)* is; in stage 3, knowing price, the firm chooses its output to maximize profits. Thus, the firm has the following profit function:

$\pi = 0$ if it does not enter; $\quad \pi^*(p) = pq^* - c(q^*) - F; \quad q^* = Arg\max(pq - c(q))$ if it does enter.

For simplicity, let $c(q) = \left(\lambda q^2/2\right)$

- **(a)** How does the price uncertainty affect the firm's (ex ante) expected profits? If the firm is risk neutral, how does the price uncertaintyt affect the maximum amount the firm would be willing to pay to enter the industry?

(b) For the particular cost function given, calculate the firm's expected profits in terms of expected price and the variance of price.

(c) Suppose the firm is risk averse, with Bernoulli utility function $u(w)$. The firm has initial wealth w_0, so if it does not enter its utility will be $u(w_0)$. If the firm enters its utility will be $u\left(w_0 + \pi^*(p)\right)$ so, from an ex ante perspective, expected utility if the firm enters will be: $\int_p u\left(w_0 + \pi^*(p)\right) dF(p)$. How does the price uncertainty affect the firm's expected utility if it enters, and thus the likelihood that the price uncertainty encourages firm entry?

(d) Suppose the distribution of price is normal with mean $\overline{p}$ and variance σ^2. Assume the individual has constant absolute risk aversion so: $u(w) = -e^{-\lambda w}, \quad \lambda > 0$. How does an increase in the variance of price affected expected utility?

6. A monopolist faces the (inverse) demand function $p(x) = a + \varepsilon - x$, where p is the price paid by consumers, x is amount of output produced by the monopolist, and a and ε are demand parameters. The (constant) unit cost of production is c, where $0 < c < a$.

(a) Suppose first that $\varepsilon = 0$. Set up and solve the profit maximization problem of the **quantity-setting** monopolist.

(b) Now suppose that $\varepsilon \in [-a, \infty)$ is a zero-mean random variable with continuous distribution function $F(\varepsilon)$, so that the monopolist is operating under demand uncertainty. The monopolist maximizes her expected utility and has a strictly concave Bernoulli utility function $u(\pi)$, where π is the profit from selling her product.

(i) Set up the monopolist's **quantity-setting** problem under this demand uncertainty assuming output is chosen before demand is known and derive the optimality condition(s).

(ii) Compare the risk-averse solution under demand uncertainty with the profit-maximizing choice under certainty (you must derive your result explicitly). Let x^0 denote the quantity produced under the optimal quantity-setting behavior of part (a) and let x^* denote the quantity produced under the optimal quantity-setting behavior of part (b)(i). Is x^* greater than or lower than x^0?

(c) Use the same assumption on demand as above, but assume the monopolist sets price (rather than quantity), and is obligated to meet realized demand at the predetermined price. For this structure, it is convenient to rewrite the demand function in direct form as $x(p) = a + \varepsilon - p$.

(i) Set and solve the problems of the **price-setting** monopolist for $\varepsilon \equiv 0$ and compare to (a).
(ii) Assume that demand is uncertain ($\varepsilon \not\equiv 0$). Assuming the firm must set price before demand is known, find how the uncertainty affects the price set by the firm, and compare to your answer to (b). Under which regime is expected output higher?