

Problem Set No. 11 Due by: Friday, December 4, 2009

1. Finish Problem Set 10
2. [Monopoly and Price Regulation]. Consider a good,  $y$ , produced by a monopolist. Consumer preferences for  $y$  are given by the quasi-linear utility function:

$$U = m + 2Aq^{1/2}y^{1/2},$$

where  $m$  is the numeraire good. “ $q$ ” denotes the quality of the good, and  $y$  the quantity of the good consumed (all goods are of the same quality). Assume “income” is large enough so the solution is always interior.

- (a) Find the consumer’s demand for good  $y$  as a function of price and quality. Also, express the inverse demand  $p(y; q)$  as a function of quality and quantity.

Next suppose the good can be produced with the following cost function:

$$C(q, y) = (1 + q^2)y; \quad q \geq 0; \quad y \geq 0$$

- (b) Find the socially optimal quantity and quality of the good (i.e.,  $\text{Max}_{q, y} \{m + 2A(qy)^{1/2} - (1 + q^2)y\}$ )
- (c) Suppose quality is fixed at  $q = 1$ . Find the profit-maximizing output of the monopolist (given the fixed quality) and compare to the socially optimal level.
- (d) Assume the government establishes a price ceiling  $\bar{P}$  on the price the monopolist can charge. Given the price ceiling, the monopolist chooses price and output  $(p, y)$  to maximize profits subject to the constraints:  $p \leq \bar{P}$  and  $y \leq D(p; q)$  where  $D(p, q)$  is the demand for the monopolist’s output as a function of price and quality. **Throughout assume quality is fixed at  $q=1$ .**
  - i. Find the monopolist’s profit maximizing solution  $y^M(\bar{P}, q)$ ,  $p^M(\bar{P}, q)$  at  $q=1$
  - ii. What happens to  $y^M$  as  $\bar{P}$  increases? Does the price control raise or lower economic welfare?
- (e) Next, assume that the monopolist is free to choose both **quality and quantity**. **Find the profit-maximizing solution**  $y^M, q^M$  (there are no price controls).
- (f) Finally, assume that, given that the monopolist is free to choose quality and quantity, the government establishes a price ceiling  $\bar{P}$ . Because the government cannot objectively measure quality, **this price ceiling does not depend on  $q$** . Given the price ceiling, the monopolist chooses  $\{p, y, q\}$  to maximize profits, subject to the constraints:  $p \leq \bar{P}$  and  $y \leq D(p, q)$  (note that demand for output depends on quality).
  - i. Find the monopolist’s profit maximizing solution  $\tilde{y}^M(\bar{P})$ ,  $\tilde{p}^M(\bar{P})$ ,  $\tilde{q}^M(\bar{P})$ .
  - ii. What happens to  $y^M$  and  $q^M$  as  $\bar{P}$  increases? Does the price control raise or lower economic welfare? Contrast your results to part (d).