1. Answer all parts.

a) A firm produces a single good, $q$, with production function $F(x_1, x_2, x_3)$. Input prices are given by $\{w_1, w_2, w_3\}$. In the short run, $x_3$ is fixed, so the firm’s short run cost curve is found from:

$$C'(q, x_3, \bar{w}) = \min \left\{ \sum_{i=1}^{3} w_i x_i \right\} \text{ s.t. } F(x_1, x_2, x_3) \geq q$$

The long run cost curve is derived from:

$$C^L(q, \bar{w}) = \min \left\{ \sum_{i=1}^{3} w_i x_i \right\} \text{ s.t. } F(x_1, x_2, x_3) \geq q;$$

The firm is a competitive profit-maximizer and sells its output at price $p$.

i. State, and prove, the relationship between short-run and long-run marginal cost. \hspace{1cm} (6 points)

ii. Assume the production function has the form: $q = F(\theta(x_1, x_2), x_3)$, where $\theta(\cdot)$ is concave and homogeneous of degree one in $\{x_1, x_2\}$, and $F(\cdot)$ is strictly concave in $\{\theta, x_3\}$. Will an increase in $p$ increase the demand for $x_1$ more in the short run or long run? Relate your answer to the properties of the function $F$ (you may assume all inputs are normal). \hspace{1cm} (8 points)

b) Consider an industry composed of identical firms, each of which has technology:

$$q = 2^{1/2} \left\{ \max \left[ (x_1 - 4), 0 \right] \right\}^{1/4} \cdot x_2^{1/4}$$

Let $\{w_1, w_2\}$ denote factor prices. In the long run there is free entry so the number of firms ($N$) is endogenous, and profits are zero. All firms are perfect competitors. For the industry, the price of input 1 ($w_1$) is exogenously determined, whereas the equilibrium price of input 2 ($w_2$) is endogenously determined, by equating industry demand for input 2 to supply, which is given by:

$$x_2^* = Aw_2$$

i. Find the long run industry supply curve for good $q$. \hspace{1cm} (8 points)

ii. What does the area next to this industry supply curve represent? Explain. \hspace{1cm} (6 points)

iii. What happens to optimal firm size as industry output expands (as you move out the industry supply curve)? \hspace{1cm} (5 points)

2. Consider a good ($x$) that can be produced in varying quality (denoted $\theta$). All consumers of the good are identical, with utility depending upon consumption of a numeraire good ($m$), and the quantity ($x$) and quality ($\theta$) of good $x$. Preferences are quasi-linear and are given by:

$$U(m, x; \theta) = m + (2 + \theta)x - \left( x^2 / 2 \right)$$
There is a single producer of this good, whose costs depend on output \((q)\) and quality \((\theta)\):

\[
C(\theta, q) = \left(\frac{q^2}{2}\right) + \left(\frac{\theta^2}{2}\right); \quad \theta \geq 0
\]

The cost function indicates that there is a fixed cost associated with developing higher quality goods, but marginal costs are independent of quality.

a) For parts (a) and (b), assume quality is exogenous and \(\theta = 1\). Find the pure monopoly solution and identify the inefficiency, if any, associated with the solution. \{**For simplicity, throughout the question you may assume there is only one consumer, so \(q = x\)**\} \(8\) points

b) Given quality as exogenous, name several policies the government can implement to increase efficiency? If the government uses a price ceiling, what is the optimal price ceiling? \(6\) points

c) Return to the pure monopoly problem (with no government regulation). Assuming quality and quantity are choice variables, find the monopolist’s profit-maximizing solution for quality and quantity. \(8\) points

d) As in part (b), assume the government imposes a price ceiling on the monopolist’s output, but the price ceiling cannot depend on quality. Assuming quality and quantity are endogenous (as in part c), find the monopolist’s profit-maximizing quality and quantity as **functions of the price ceiling**. What is the optimal price ceiling? (Decisions are made in the following sequence: (1) the government sets the price ceiling, which cannot depend on quality; (2) the monopolist chooses quality and output; (3) the consumer decides how much to buy, subject to availability). \(11\) points

3. Answer all parts.

a) A cost-minimizing firm has cost function \(C(q; \tilde{w}) = \tilde{q}^h \cdot \{w_3 \cdot \text{Min}(w_1, w_2)\}^{1/2}\).

i. Derive the quasi-concave production function dual to this cost-curve. Are there non-quasi-concave functions that could have generated this cost function? If so, give an example. \(10\) points

b) Suppose an individual has the utility function: \(U = H(\phi(x_1, x_2), \theta(x_3, x_4))\) where \(\theta, \phi, H\) are all increasing functions of their arguments.

i. If \(\phi\) and \(\theta\) are both homogeneous of degree one functions, what can you conclude about how an increase in the price of good 1 \((p_1)\) affects the utility maximizing demands for \(x_3\) and \(x_4\)? How will it affect the ratio of demands \((x_3/x_4)\)? Be as specific as possible. \(5\) points

ii. How does your answer to part (i) change for the special case when: \(H(\phi(x_1, x_2), \theta(x_3, x_4)) = \phi(x_1, x_2) \cdot \theta(x_3, x_4)\)? \(7\) points

c) An individual has Hicksian demand for good 2 given by: \(x_2^h = u \cdot (p_1/p_2)^{1/2}\). Initially the individual faces the price vector \((p_1, p_2) = (1, 9)\). The individual has the opportunity to purchase good 2 at a price \(p_2 = 1\) but he must buy at least 10 units of the good (which cannot be resold). What is the maximum amount the person would pay for the right to shop at this lower price (given the quantity constraint)? Your answer should depend on \(u\). \(11\) points
4. Answer all parts.

a) An individual with wealth $W_0$ faces a probability $\pi$ of becoming sick. If the person is sick she suffers a monetary loss of $L$ (lost income, medical bills, etc). The person can buy insurance to reimburse for a fraction $\alpha$ of the loss, so in the event a loss occurs she receives $(\alpha L)$ from the insurance company. Insurance costs $q$ per dollar of insurance, so the cost of the insurance policy is $(q\alpha L)$. Hence, realized wealth is:

$$W = \begin{cases} W_0 - L(1 - \alpha) - q\alpha L & \text{with probability } \pi \\ W_0 - q\alpha L & \text{with probability } (1 - \pi) \end{cases}$$

Let the person’s Bernoulli utility function be $u(W)$, with $0 < u' > 0 > u''$.

i. Write down the person’s expected utility. (4 points)

ii. Show how the person determines the optimal amount of insurance to buy (determines $\alpha$). If $q > \pi$ will the person buy full insurance (will $\alpha = 1$)? (6 points)

iii. How will an increase in $L$ affect the optimal choice of $\alpha$? Prove your answer. (You may assume non-increasing absolute risk aversion). (6 points)

(b) Consider a competitive firm which has the following production function: $Q = 2K^{1/4}L^{1/2}$. Input prices are, as usual, $(W, R)$) and the output price is $p$. When the firm makes its investment decision (chooses $K$) price is unknown, but when it chooses $L$ price ($p$) is known (you can think of investment as a long run decision and labor as a short run decision). Hence, decisions are made in the following sequence: (1) $K$ is chosen; (2) the firm learns the true value of $p$; and then (3) the firm chooses its profit-maximizing value of $L$. Assume the firm is risk-neutral and, when choosing $K$, maximizes expected profits. The ex ante distribution of $p$ is given by $F(p)$, with mean $\bar{p}$ and variance $\sigma_p^2 > 0$. {As always, problems like this are solved backwards}.

i. Given $K$, $p$, find the optimal value of $L$. (5 points)

ii. Use the restricted profit function $\pi(p, K; W, R)$ from (i) to find the optimal investment decision, assuming the firm is risk neutral and maximizes expected profits. (7 points)

iii. How does the price uncertainty affect the firm’s optimal investment level and expected profits (compared to the certainty case where price is always equal to its expected value?) Be specific. (5 points)
Consider a simple “general equilibrium” model of an economy with three goods, two consumers and two firms (all consumer and firms act as competitive price takers, so essentially there a large number of each type of consumer and firm). Consumer preferences and producer technology are:

\[ U^h = m^h + \alpha^h_1 x^h_1 + \alpha^h_2 x^h_2 - \frac{(x^h_1)^2}{2} - \frac{(x^h_2)^2}{2} + \gamma x^h_1 x^h_2; \quad |\gamma| < 1; \quad \alpha^h_i > 0, \quad h = 1, 2; \quad i = 1, 2; \quad \alpha^h_i > 0 \]

\[ C^j = \frac{\beta^j_1 (q^j_1)^2}{2} + \frac{\beta^j_2 (q^j_2)^2}{2} + \delta (q^j_1 \cdot q^j_2); \quad \beta^j_i > 0; \quad [\beta^j_1 \beta^j_2 - (\delta)^2] > 0 \]

(In other words, preferences are strictly quasi-concave and the cost function is convex). Good \( m \) is the numeraire, with price normalized to 1, so the prices of the other goods are relative prices. \( C^j (q^j_1, q^j_2) \) is the cost, in terms of the numeraire, of producing the given output vector. Each consumer is endowed with \( M^h \) units of the numeraire, and firm profits are redistributed back equally to consumers. You may assume throughout that all solutions are interior.

a) Suppose the government decides to help consumer 1 by subsidizing his purchases of good 1 (the subsidy is \( s^1 \) per unit purchased); the subsidy is paid for by a lump sum tax on consumer 2. Derive the demand curves for each consumer and the supply curves for each firm, and solve for equilibrium prices as a function of the subsidy. (7 points)

i. What inefficiency is (inefficiencies are) caused by this subsidy? Be specific. (5 points)

ii. Can you use the supply and demand curves for good 1 to measure the deadweight loss due to the subsidy? Why or why not? (4 points)

b) Modify the assumption of part (b) and assume the government gives both (all) consumers the same per unit subsidy for good 1 (denoted \( s^1 \)). For political reasons the government cannot eliminate this subsidy, but it is contemplating a tax (or subsidy) on good 2 – denote the tax by \( t_2 \) (if \( t_2 \) is negative then it is a subsidy). If there is a budget deficit (or surplus), then it is financed by lump sum taxes on (transfers to) consumers – meaning, in essence, you do not need to model the deficit. For this part assume \( \gamma = 0 \).

i. For \( t_2 \neq 0 \), can you measure the deadweight loss of the subsidy using the area next to the supply and demand curves for good 1? Similarly, for \( s^1 \neq 0 \), can you measure the deadweight loss from the tax by the area next to the demand curves for good 2? Why or why not? (5 points)

ii. Given \( s^1 > 0 \) find the optimal tax or subsidy on good 2. (12 points)