1. Consider a monopoly firm with cost curve: \( C(q) = q \), where costs are measured in terms of the numeraire good. The firm has two types of customers, with quasi-linear preferences as follows:

Type 1: \( U(m, x_1) = m_1 + 10x_1 - \left( \frac{x_1^2}{2} \right) \);  
Type 2: \( U(m, x_1) = m_2 + 16x_2 - \left( \frac{x_2^2}{2} \right) \);

There are an equal number of each type of customer. In the above, \( (m_h, x_h), h = 1, 2 \) is the consumption vector of customer of type \( h \), where \( m \) is the numeraire – whose price is one – and \( x \) is the monopolist’s product.

a) Consider the case of pure monopoly, in which the firm must charge the same price for all customers, and must allow customers to choose how much to buy at that price. Find the profit maximizing solution for the monopolist, and calculate the inefficiency (deadweight loss) due to the monopoly. (the monopolist’s output must satisfy: \( q \geq (x_1 + x_2) \)). (10 points)

i. Suppose the monopolist, in addition to charging each customer the same price, can charge each customer a fixed fee \( F \) to shop at his store. However, this fixed fee must be the same for all customers. **Find the optimal price and fixed fee** (check the participation constraint) and compare to the solution when no fee can be charged. (re-sale is not allowed). (11 points)

b) **Second degree price discrimination.** Suppose the monopolist can offer two different packages. Package A offers \( x^A \) units at a total cost to the buyer of \( r^A \), and package B offers \( x^B \) units at a total cost of \( r^B \). Either type of consumer can choose either package. Find the profit-maximizing solution for the monopolist and compare to both solutions from part (a). (12 points)

2. Answer all parts.

a) Consider a competitive industry in which all (actual and potential) firms have the same cost function:

\[
C(w_1, w_2, w_3, q) = \left( q^{1/2} w_1 + q^{3/2} \left( w_2^{1/2} w_3^{1/2} \right) \right)
\]

where \( q \) denotes the firm’s output and \( w_1, w_2, w_3 \) are factor prices. Let \( D(p, A) = Ap^{-1/2} \) denote the demand for the industry’s output, where \( p \) is price and \( A \) is a demand parameter (assume solutions are interior). The industry long-run equilibrium is characterized by \( (p^*, q^*, J^*) \) where \( p^* \) is equilibrium price, \( q^* \) is equilibrium output per firm and \( J^* \) is the number of (identical) firms (you may treat \( J \) as a continuous variable,).

i. Assuming input prices are exogenous, derive the long run industry supply curve and find the equilibrium values of \( (p^*, q^*, J^*) \). (8 points)
ii. Assume input prices $w_2, w_3$ are exogenous, but the industry faces a positively sloped supply of input 1, given by $S(w_1) = Bw_1$. Find the long run industry supply curve (i.e., find $p^*(Q)$, where $Q$ is aggregate industry supply). What does the area next to this supply curve represent? Be as specific as possible. (9 points)

b) Suppose a firm uses three inputs to produce a single good, $q$. An econometric estimate indicates the cost function has the following form:

$$C(q, w_1, w_2, w_3) = q^2 \cdot \left[ w_1^\alpha w_2^\beta + w_3^\gamma \right]^\delta$$

i. What restrictions must hold on the parameters $\alpha, \beta, \gamma, \delta$ for this to be a legitimate cost function? Explain your answer. (7 Points)

ii. Let $\delta = 2$, $\alpha = \beta$, and assume the other parameters are such that the function $C(q, \bar{w})$ obeys all the properties of a cost function. Find the production function dual to $C(q, \bar{w})$. (9 points)

3. Answer all parts.

An individual with wealth $W_0$ owns a house. With probability $\pi$ the house will be damaged or destroyed (by fire or floods), imposing a loss of $L$ on the homeowner. The homeowner can protect himself against this loss in two different ways:

1) **By safety measures**: the amount of the loss, $L(P)$, depends on preventive measures ($P$), such as fire sprinklers or waterproofing: $\left( dL(P)/dP \right) < 0 < \left( d^2L(P)/dP^2 \right)$

2) **Buy house insurance**: the person can choose how much insurance ($I$) to buy. Insurance pays $I$ dollars if a loss occurs; the person pays $\alpha I$ for this insurance, whether or not there is a loss. Hence, realized wealth for the individual is:

$$W = \begin{cases} 
(W_0 - (\alpha I + P) - (L(P) - I)) & \text{with probability } \pi \\
(W_0 - (\alpha I + P)) & \text{with probability } (1 - \pi)
\end{cases}$$

Note that $W_H > W_L$ if $L > I$. Let the person’s utility function be $u(W)$, with $u' > 0 > u''$.

a) For part (a) **assume house insurance is not available**, so the only choice is how much to spend on preventive measures.

i. Write down the person’s expected utility and derive the FOC for the optimal amount to spend on loss prevention. Is the SOC satisfied? (assume throughout an interior solution). (8 points)

ii. Does a risk averse person spend more on loss prevention ($P$) than a risk neutral person, or does s/he spend less? Prove your answer. (5 points)

iii. How does an increase in $W_0$ (the person’s wealth) affect the optimal choice of $P$? Relate your answer to the measure of absolute risk aversion. (5 points)
b) Now assume that the person can buy insurance, as well as undertake preventive measures.

i. Write down the FOCs for both $I$ and $P$. (5 points)

ii. Assume the insurance is “actuarially fair” (i.e., $\alpha = \pi$). Under what conditions will the person still spend money on prevention? (5 points)

iii. Finally, assuming both $I > 0$ and $P > 0$ at the solution, how does an increase in insurance rates affect the optimal amount of prevention? (i.e., find the sign of $(dP^*/d\alpha)$) (5 points)

4. Answer all parts.

a) Assume a representative individual with quasi-linear preferences: $U(m, c) = m + 100c^{1/4}$.

As usual, $m$ is the numeraire good and $c$ represents consumption of a good produced by a natural monopolist. Assume the monopolist has the following total cost function: $C(q) = (5/2)q^{3/4}$ where, of course, cost is measured in terms of the numeraire.

i. Assume the only policy tool available to the government is a price ceiling. Assuming the firm will not produce if it loses money, find the (constrained) optimal price ceiling. (7 points)

ii. Assume the government can tax or subsidize the firm’s output. Determine the tax/subsidy which maximizes welfare (the sum of producer surplus, consumer surplus and net tax revenue). (9 points)

b) Consider a consumer whose utility depends on three goods, and who has expenditure function $e(p_1, p_2, p_3, U)$. The person’s plan is to allocate spending over her lifetime to maintain a constant level of utility $U^0$ (and leave whatever assets remain to her children). Prices have been stable at $(p_1^0, p_2^0, p_3^0)$ and the person has purchased the (stable) bundle $(x_1^0, x_2^0, x_3^0)$. Suppose that unexpectedly the price of good one increases so that the new (permanent) price vector is $(p_1^1, p_2^0, p_3^0)$, with $p_1^1 > p_1^0$. Also assume that good 3 is housing, and that this cannot be changed in the short run; however, given enough time (i.e., the long run), the person will adjust purchases of housing (and the other goods) to the new optimal level.

i. Compare the short run and long run effects of this price increase on (1) the expenditures of the individual; (2) the demand for good 1 and (3) the demand for good 2. (8 points)

ii. Assume the expenditure function is given by: $e(\bar{p}, U) = U \cdot \left[ H(p_1, p_3), p_2 \right]$, where $H(p_1, p_3)$ is a concave, homogeneous of degree one function of its arguments. In the long run, do purchases of good 3 rise or fall? Also, compare the short run and long run impact of the increase in $p_1$ on the demand for good 2. (9 points)
5. Answer all parts.

Consider a “simple” general equilibrium model of an economy. One good, the numeraire, is used to produce the other two goods. There are \( J \) identical firms, each with the same cost function:

\[
C^j(q^j_1, q^j_2) = (\lambda/2) \left[ (q^j_1)^2 + \theta q^j_1 q^j_2 + (q^j_2)^2 \right]; \quad |\theta| < 2; \quad j=1,\ldots,J
\]

\((q^j_1, q^j_2)\) is the output vector of firm \( j \), and \( C^j \) is the amount of the numeraire good required to produce that output vector. In addition, output of good one generates a byproduct, called pollution \((Z)\), which may harm households:

\[
Z = \beta \cdot (\sum_j q^j_1), \quad \beta > 0
\]

Household welfare depends on consumption of the numeraire and the two produced goods, and on the level of pollution. There are two types of households (and \( J \) households of each type), with the following preferences:

\[
U^h = m^h + \alpha^h_1 x^h_1 + \alpha^h_2 x^h_2 - \frac{(x^h_1)^2}{2} - \frac{(x^h_2)^2}{2} - \delta Z^2; \quad \delta \geq 0; \quad \alpha^h_1 > \alpha^h_2 > 0, \quad i=1,2;
\]

In the above, \((m^h, x^h_1, x^h_2)\) is consumption by household of type \( h \) (=1,2) of the numeraire good \((m)\) and the two produced goods \((i=1,2)\), and \((\delta Z^2)\) measures the impact of pollution on household utility. Note that all households of the same type have the same endowment and preferences.

Each consumer is endowed with \( M^h \) units of the numeraire, and firm profits are redistributed back equally to consumers. **You may assume throughout that all solutions are interior.**

a) Find the \textit{laissez-faire} competitive equilibrium for this economy (i.e., there is no government policy). Show that it is \textit{pareto efficient} if, and only if, \( \delta = 0 \). \hspace{1cm} (10 points)

b) Assume \( \delta > 0 \). **Discuss** the optimal government policy to correct the market failure. \hspace{1cm} (5 points)

i. Find the optimal value of this government policy and show how it changes as the size of the economy (as \( J \)) increases. \hspace{1cm} (7 points)

c) Suppose the only feasible government policy is to tax or subsidize production (consumption) of good 2. **Discuss** whether the second-best policy is a tax or subsidy and relate your answer to the sign of \( \theta \). \hspace{1cm} (5 points)

i. Find the optimal second-best policy (i.e., the optimal tax or subsidy to good 2). \hspace{1cm} (6 points)