Economics 601, Microeconomic Analysis I  
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Midterm 2

1. Answer all parts.

(a) Consider a consumer whose utility depends on three goods, \( \{ x_1, x_2, x_3 \} \). Her Hicksian demand for goods 1 and 2 are given by:

\[
x_1^h = U^{1/2} \left( \frac{P_2^{1/3} P_3^{1/6}}{P_1^{1/2}} \right), \quad x_2^h = \frac{2U^{1/2}}{3} \left( \frac{P_3^{3/2} P_1^{1/6}}{P_2^{3/3}} \right),
\]

where \( U \) is utility, and \( p_i \) is the price of good \( i \). Currently, the individual has income \( w = 72 \), prices are \((9, 1, 1)\) and the person’s utility-maximizing consumption bundle is: \((x_1, x_2, x_3) = (4, 24, 12)\).

i. A new discount store opens that allows her to buy \( x_1 \) at a price of \( p_1 = 4 \) per unit. What is the maximum amount she would pay for the opportunity to shop at this store? \( (5 \text{ points}) \)

ii. Suppose the discount store (from part (i)) requires individuals to purchase at least 12 units (at \( p_1 = 4 \)). Now how much will the person pay for the right to shop at the store? \( (7 \text{ points}) \)

(b) Consider an individual with the Hicksian demands given in part (a), who currently faces prices \((9, 1, 1)\) and chooses the utility maximizing bundle \((4, 24, 12)\). If the person’s income is unchanged, but prices change to \((4,27/8,1)\), will the person be better off or worse off? A precise answer is required. \( (7 \text{ points}) \)

(c) Consider an individual with the following utility function: \( U = \phi \left( H \left( m_1, m_2, m_3 \right), c \right) \), where \( H \) is a strictly increasing, concave and homogeneous of degree one function of \((m_1, m_2, m_3)\) and \( \phi(\ldots) \) is a strictly quasi-concave, homothetic function of \( H \) and \( c \). Let prices be given by \((p_{m1}, p_{m2}, p_{m3}, p_c)\) and income by \( w \). Answer the following questions assuming the individual maximizes utility.

i. How will an increase in \( p_c \) affect demands for \((m_1, m_2, m_3)\) and how will it affect the ratio of demands \((m_2/m_1)\) and \((m_3/m_1)\)? Explain your answer. \( (6 \text{ points}) \)

ii. Let \( H \left( m_1, m_2, m_3 \right) = \left( m_1 m_2 m_3 \right)^{1/3} \). Suppose the price vector changes from \( \{p_{m1} = 4; \ p_{m2} = p_{m3} = p_c = 1\} \) to \( \{p_{m2} = p_{m3} = 2; \ p_{m1} = p_c = 1\} \), while income is unchanged. How will this affect the demands for the 4 goods? Be precise. \( (8 \text{ points}) \)
2. Answer all parts.

(a) Consider a single product firm which uses inputs of $\bar{x} \in \mathbb{R}^L_+$ to produce output $q \in \mathbb{R}^1$; the firm’s production technology is given by $q \leq f(\bar{x})$. The production set for the firm is given by:

$$ Y = \{(q, -x_1, \ldots, -x_L) \in \mathbb{R}^{L+1} \mid q - f(\bar{x}) \leq 0, \text{ and } \bar{x} \geq 0\} $$

i. Using either the production function or the production set, define strictly decreasing returns to scale (DRS). Does DRS imply the firm’s production function is concave? What does DRS imply about the firm’s cost function? Prove your answer. (5 points)

ii. Define the firm’s cost function:

$$ C(q, \bar{w}) = \min_{\bar{x}} \{\bar{w} \cdot \bar{x} \mid f(\bar{x}) \geq q\}. $$

Prove that (strict) concavity of the production function implies the cost curve is (strictly) convex in $q$. (6 points)

(b) For the remainder of this question assume that the production function is given by:

$$ q \leq f(x, \lambda) = \left(\lambda^1 + (2x_2)^4\right)^{1/4}; \quad \lambda > 0 $$

i. Derive the firm’s cost function. (7 points)

ii. Is the production function concave? Is the cost function convex in $q$? Under what conditions will there be a solution to the firm’s profit maximization problem? (4 points)

iii. Using the cost function derived above, set up and solve the profit-maximization problem, clearly indicating under what conditions a solution exists. (6 points)

iv. Suppose you had set up the profit maximization problem without first deriving the cost function (i.e., you solve $\max_{x_1, x_2} \left\{ p \left(\lambda^1 + (2x_2)^4\right)^{1/4} - w^1 x_1 - w^2 x_2 \right\}$). For those cases where a solution exists, would you get the same solution as in part (iii)? What difficulties, if any, might you encounter in finding the solution? (you do not have to actually solve the problem this way). (5 points)
3. A competitive profit-maximizing firm produces a good, $q$, using a monotonic increasing, concave three input production function $f(x_1, x_2, x_3)$. Let $p$ denote output prices, and $(w_1, w_2, w_3)$ denote input prices. Let $\pi(p, w_1, w_2, w_3)$, the firm’s (maximum) profit function, be given by:

$$\pi(p, w_1, w_2, w_3) = \frac{p^2 (W_1 + W_2)^{1/2}}{(W_1 W_2 W_3)^{1/2}}$$

(a) Find the firm’s profit maximizing supply and factor demand functions. \hspace{1cm} (8 points)

(b) Find the firm’s cost function. How does an increase in $W_1$ affect the profit-maximizing demand for $x_3$ and how does this increase in $W_1$ affect the conditional factor demand (the cost-minimizing demand) for $x_3$. Explain the differences, if any, between these results. \hspace{1cm} (6 points)

(c) Find the production function dual to this profit function. \hspace{1cm} (12 points)

(d) Suppose that, in the short run $x_1$ and $x_2$ are choice variables and $x_3$ is fixed, while in the long run all three inputs are choice variables. Let $\bar{x}_3$ denote the fixed level of input 3, and let $\Omega$ denote the set of prices so that the optimal long run choice equals $\bar{x}_3$. Using the profit (or production) function, what can you conclude about the relationship between the short run and long run values of $\left(\frac{\partial q}{\partial p}, \frac{\partial x_1}{\partial p}, \frac{\partial x_2}{\partial p}\right)$, when evaluated for prices in $\Omega$. Explain your results. \{Use all the information embodied in the profit function; do not just provide a generic answer\}. \hspace{1cm} (7 points)
4. Answer all parts.

a) Consider a good produced using two inputs: \( q \leq f(x_1, x_2) \). Let \( \hat{x}_i(q, w_1, w_2), i = 1, 2 \), denote the cost-minimizing (or conditional) factor demands, and \( x_i^*(p, w_1, w_2), i = 1, 2 \), denote the profit-maximizing factor demands (assume the profit-maximizing solution exists). What is the sign of \( \frac{\partial \hat{x}_1}{\partial w_2} \)? Under what conditions can the sign of \( \frac{\partial \hat{x}_1}{\partial w_2} \) be different than the sign of \( \frac{\partial x_1^*}{\partial w_2} \)? Be specific and justify your answer. (7 points)

b) Consider a firm which produces a good \( q \) using two inputs. The profit-maximizing input demands are:
\[
\begin{align*}
\hat{x}_1^* &= \frac{4p^2}{W_1^{4/3}W_2^{2/3}}, \\
\hat{x}_2^* &= \frac{8p^2}{W_1^{4/3}W_2^{2/3}}.
\end{align*}
\]
Output price is \( p = 27 \), and input prices at the closest store are: \( W_1 = W_2 = 27 \).

i. Suppose a store in a neighboring town sells inputs at the prices \( W_1 = 8; \ W_2 = 27 \). The monetary cost of making the trip to this store is \( F_1 \). For what values of \( F_1 \) will the firm choose to make the trip? If it does, calculate the change in the firm’s optimal input purchases and in the firm’s profit maximizing output supply. A precise answer is required. (10 points)

c) Consider an individual with the utility function \( U = c \cdot l \), where \( c \) is consumption and \( l \) is leisure. The individual is endowed with \( T = 24 \) units of time, which is split between labor \( (L) \) and leisure \( (l) \). The individual has a non-labor source of income, \( I_0 \), which, for example, comes from investment income \( (I_0 \) can be negative). Finally, the person has to pay tax (only) on earned income; the tax rate is \( \tau_1 = (1/4) \) for earned income less than \( 100 \), while the (marginal) tax rate on earnings above \( 100 \) is \( \tau_2 = (1/2) \). Thus, the budget and time constraints are:
\[
\begin{align*}
[WL(1-\tau_1)+I_0- pc] &= [(3W/4)L+I_0- pc] \geq 0 \quad &\text{if } &WL \leq 100 \\
[WL-\tau_1100-\tau_2(WL-100)+I_0- pc] &= [(W/2)L+25+I_0- pc] \geq 0 \quad &\text{if } &WL \geq 100 \\
[24-l-L] &\geq 0 \quad &\text{where: } &p=1, \ \ W = 20
\end{align*}
\]
i. Draw the budget constraint facing the individual, paying careful attention to the change in tax rates. Is the budget set convex? Is the boundary of the budget set linear? Explain. (6 points)

ii. Find the individual’s optimal decisions \( (l^*, L^*, c^*) \) as a function of \( I_0 \); pay careful attention to how the change in the marginal tax rate affects the consumer’s decisions. Graph the utility-maximizing labor supply as a function of “investment income” \( I_0 \). (10 points)