2.1 If $\succeq$ is a rational preference relation, prove that $x_3 \succeq x_2$, $x_2 \succ x_1 \rightarrow x_3 \succ x_1$. If $x_3 \succeq x_2$ and $x_3 \succ x_1$, what can you infer about $x_2$ as compared to $x_1$?

2.2 Consider the properties of local non-satiation, monotone and strongly monotone {MGW (p.42)}
(a) Does local non-satiation imply monotone preferences? Do monotone preferences imply local non-satiation? Prove your answer.
(b) Repeat (a) for the relationship between monotone and strongly monotone preferences.

2.3 Jehle and Reny define convexity (strict convexity) of preferences as follows:
If $x^i \succeq x^0$, then $tx^i + (1-t)x^0 \succeq x^0, t \in [0,1]$ {convexity}
If $x^i \succeq x^0$, $x^i \neq x^0$, then $tx^i + (1-t)x^0 \succ x^0, t \in (0,1)$ {strict convexity}

MGW define convexity (strict convexity) of preferences as follows:
If $y \succeq x$, and $z \succeq x$, then $ty + (1-t)z \succeq x, t \in [0,1]$ {convexity}
If $y \succeq x$, and $z \succeq x$, $y \neq z$, then $ty + (1-t)z \succ x, t \in (0,1)$ {strict convexity}

(a) Assuming preferences are rational, are these definitions equivalent? Prove your answer.
(b) Given an illustration of a utility function that represents preferences which are convex, but not strictly convex.

2.4 Consider the following preferences over consumption bundles in $\mathbb{R}^2$.
$x \succeq y \iff (x_1, x_2) > (y_1, y_2)$ OR if $x_1, x_2 = (y_1, y_2)$, then $x_i \succeq y_i$

Note that $x_i$ denotes the amount of good $i$ in bundle $x$.
(a) Are these preferences rational (complete, transitive)?
(b) Are they consistent with the monotonicity assumptions? The convexity assumptions?
(c) Draw the upper and lower contour sets to a given bundle $x^0$.
(d) Does these preferences satisfy the continuity assumption?
(e) Given the budget constraint, is there a solution to the utility maximizing problem?

2.5 A consumer with preferences represented by differentiable utility function $u(x, y)$ faces the budget constraint $p_x x + p_y y = w$, where $w$ is income. For the following situations indicate if the given consumption bundle is a local maximum and if it is a global maximum. If it is not a local maximum, indicate how the consumption bundle can be altered to increase utility. Also indicate the role convexity plays in your answer. (The notation $u_j$ indicates partial differentiation with respect to good $j$).

(a) $x = 0$, $y = (w/p_y)$, $(u_x/u_y) > (p_x/p_y)$
(b) $x = 0$, $y = (w/p_y), (w/p_x)$ $(u_x/u_y) \leq (p_x/p_y)$
(c) $x > 0, y > 0; (u_x/u_y) < (p_x/p_y)$
(d) $x > 0, y > 0; (u_x/u_y) = (p_x/p_y)$
2.6 Consider the problem: 
\[
\max_{x,y} f(x, y) = 6(x + y) - \left(\frac{x - y}{2}\right)^2 \quad \text{s.t.} \quad x, y \in \mathbb{R}_+^2 \text{ and } (x + y) \leq A, A > 0
\]
(a) Is this utility function concave? Does it exhibit monotonicity?; local non-satiation? Explain.
(b) Must a solution to this maximization problem exist? Is every local maximum a global maximum?
(c) Find the solution \((x^*, y^*)\) and show how your answer depends on the value of \(A\). Do the necessary and sufficient conditions of the Kuhn-Tucker condition apply?
(d) Let \(A=20\). Assume the objective function is unchanged but the domain is now defined by:
\[
x, y \in \mathbb{R}_+^2 \text{ and } [20 - x - y]^3 \geq 0.
\]
Is this domain any different than that for the original problem with \(A=20\)? What are the values of \((x^*, y^*)\) that solve the optimization problem?
(e) Formulate the Lagrangean for the problem of part (d), with constraint \((20 - x - y)^3 \geq 0\). Is there any value of \(\lambda^*\) s.t. \((x^*, y^*, \lambda^*)\) solve the KT conditions? Explain.

2.7 Consider the problem 
\[
\max_{x,y} \left(2x^2 + y^2\right) \quad \text{s.t.} \quad x, y \in \mathbb{R}_+^2 \text{ and } (x + y) \leq 10.
\]
Since the domain is compact and the objective function is continuous, we know there is a global maximum, which we call \((x^*, y^*)\).
(a) Formulate the Lagrangean function for this problem and find all solutions to the K-T conditions. Do all solutions to these conditions solve the original maximization problem? Do all represent local maxima? Explain.
(b) What is the global maximum \((x^*, y^*)\) for the problem given in the introduction to this question? Must there be a \(\lambda^*\) such that \((x^*, y^*, \lambda^*)\) solves the K-T conditions? If so, find it.

2.8 Cobb-Douglas preferences
Recall the 2-good utility function of the Cobb-Douglas form briefly discussed in class:
\[
u(x) = x_1^\alpha x_2^{1-\alpha}, \text{ where } \alpha \in (0,1).
\]
Let the budget constraint be \((p_1x_1 + p_2x_2) \leq w, \ (p_1, p_2) \gg 0, \ w > 0\)
(a) Show this utility function is both concave and quasiconcave and derive the demand curves.
(b) Let \(v(x) = \left(x_1^\alpha x_2^{1-\alpha}\right)^d\). Is this function concave?; quasi-concave? Does this function exhibit diminishing marginal utility for both goods; for either good? What are the demands functions for this utility function?