1. A monopoly producer has the following cost function: \( C(q) = \frac{\alpha q^2}{2} \).

The market demand for the monopolist’s output is: \( q^D = 64 p^{-2} \rightarrow p = 8q^{-1/2} \)

a) Find the socially efficient output level. (6 points)

b) Assuming no government intervention, find the monopolist’s profit maximizing output and price, and identify graphically the deadweight loss due to the monopoly behavior. (7 points)

i. Suppose the government is considering subsidizing the firm to achieve the efficient output level. Let \( s \) be the per unit subsidy, so the monopolist’s profit is:

\[ \pi = (p + s)q - C(q); \quad p = 8q^{-1/2} \]

Find the optimal subsidy as a function of \( \alpha \). (5 points)

c) Suppose we modify the problem by assuming the firm can reduce its production costs, \( C(q) \), by investing in research and development, \( R \). In particular, the firm’s total costs are:

\[ TC(q) = \frac{\alpha(R)q^2}{2} + R = \frac{(1 + R)^{-1} q^2}{2} + R \]

i. Find the socially efficient research and output levels. (7 points)

ii. Suppose the government sets the subsidy after the firm conducts its research and development. Thus, decisions are made in the following order: (1)the firm invests in research; (2)given the research and resulting cost function of the firm, the government chooses the (optimal) subsidy; and (3)given \( R \), and the subsidy, the monopoly firm chooses its output level. Given this timing, derive:

(1) The optimal subsidy as a function of the firm’s research decision \( \left(s^*(R)\right) \) (4 points)

(2) The firm’s optimal research decision. How does the resulting equilibrium differ from the socially efficient solution? Give an intuitive explanation for your result. (4 points)
2. A farmer has $L$ units of land which will be used to grow corn. The price at which s/he can sell corn, $p$, is known. There are two types of seeds he can plant, a drought resistant variety that always yields $y$ bushels per acre, or a high-yield variety, which has lower yields $(x_L < y)$ in case of a drought, but higher yields $(x_H > y > x_L)$ in good weather (the “high-yield” variety has higher expected yields).

Let $L_d$ be the amount of land planted in the drought resistant variety, so that $(L - L_d)$ is the amount planted in the high yield variety. The probability of a drought is $\pi$, the cost per acre is $c$, which is the same for either variety, and the farmer’s initial wealth is $w_0$. Denote the farmer’s ex post wealth as $w$, where:

With probability $\pi$:
$$w = (w_0 - c) + p\left\{L_d y + (L - L_d)x_L\right\}$$

With probability $(1 - \pi)$:
$$w = (w_0 - c) + p\left\{L_d y + (L - L_d)x_H\right\}; \quad \pi x_L + (1 - \pi)x_H > y$$

Finally, let $u(w)$ be the farmer’s Bernoulli utility function, and assume he behaves in such a way as to maximize expected utility.

(a) Set up the farmer’s maximization problem. (6 points)

(b) Assuming the farmer is risk neutral, what is his optimal planting decision? (5 points)

(c) Assume the farmer is risk averse. Will he ever plant all acreage with the drought resistant (and hence safe) variety? Prove your answer and explain. (6 points)

For the rest of the problem, assume the farmer is risk averse and that there is an interior solution in which the farmer plants both varieties.

(d) Assume there are two farmers ($A$, $B$) who have the same amount of land, have the same preferences and face exactly the same decision. The only difference is that farmer A has more initial wealth than farmer B. Compare the decisions made by the two farmers. Be as specific as possible and prove your answer. (8 points)

(e) Assume that farmer $C$ has the same amount of wealth, the same amount of land and faces the same decision as farmer A, but that their preferences differ. Farmer A’s Bernoulli utility function is $u(w)$, $\frac{du}{dw} > 0 > \frac{d^2 u}{dw^2}$, while farmer C’s utility function is $h(w)$, where

$$h(w) = \phi\left(u(w)\right), \quad \frac{d\phi}{du} > 0 > \frac{d^2 \phi}{du^2}.$$ Compare the planting decisions made by the two farmers, and prove your answer. (8 points)
3. Answer all parts.

a) Consider a competitive industry in which all (actual and potential) firms have the same cost function:

\[ C(w_1, w_2, q) = \left( q^{4/3} w_2 + q^{1/3} \left( w_2^{1/2} w_1^{1/2} \right) \right) \]

where \( q \) denotes the firm’s output and \( w_1, w_2 \) are factor prices. Assume free entry of firms. Write the industry’s long run inverse supply curve as \( P(Q) \), and the long run equilibrium output per firm, and number of firms, as a function of industry output as \( q^*(Q) \), \( J^*(Q) \). (you may treat \( J \) as a continuous variable,).

i. Assuming input prices are exogenous, and for simplicity \( w_1 = w_2 = 1 \), derive the long run industry supply curve and find the equilibrium values of \( (p^*(Q), q^*(Q), J^*(Q)) \). (8 points)

ii. Assume \( w_2 = 1 \), but that the price of input one is endogenous. In particular, the industry faces a positively sloped supply of input 1, given by \( S(w_1) = Bw_1 \). Find the long run industry supply curve (i.e., find \( p^*(Q) \), where \( Q \) is aggregate industry supply) and the equilibrium firm size \( q^*(Q) \) as a function of total output. What does the area next to this supply curve represent? Be as specific as possible. (10 points)

b) Suppose a firm uses two inputs to produce a single good, \( q \). An econometric estimate indicates the cost function has the following form:

\[ C(q, w_1, w_2) = q^h \left( w_1^\alpha + w_2^\gamma \right)^\delta \]

i. What restrictions must hold on the parameters \( \alpha, \gamma, \delta, h \) for this to be a legitimate cost function? Explain your answer. (5 Points)

ii. Let \( \delta = -2 \), \( h = 3 \), and assume the other parameters are such that the function \( C(q, \tilde{w}) \) obeys all the properties of a cost function. Find the production function dual to \( C(q, \tilde{w}) \). (10 points)
(a) Consider an economy with \( H \) identical consumers, each of whom has the following preferences:

\[
U^h = m^h + \alpha_1 x_1^h + \alpha_2 x_2^h - \frac{(x_1^h)^2}{2} - \frac{(x_2^h)^2}{2} + \gamma x_1^h x_2^h; \quad |\gamma| < 1; \quad \alpha_i > 0, \quad i = 1, 2
\]

There are three consumption goods – the numeraire (good \( m \)), and goods 1 and 2. Let \( (p_1, p_2) \) be prices facing the consumer, where the price of good \( m \) is normalized to 1. There are a large number of identical firms who can produce either good one or good two, using the numeraire good as input. The production costs, measured in terms of the numeraire, for firm \( j \) are:

\[
C^j = \left( y_1^j + 2y_2^j \right), \quad \text{where } y_i^j \text{ is firm } j \text{'s output of good } i.
\]

Assume all tax revenue is rebated, equally, to consumers.

i. Suppose there is no tax on good 2, but an ad valorem tax at rate \( \tau_1 \) on good 1. Show mathematically how this tax affects the equilibrium quantities consumed and equilibrium consumer prices. (5 points)

ii. Calculate, and show graphically, the deadweight loss from this tax. (7 points)

iii. Assume \( \tau_1 > 0 \) and that the government cannot change this tax. The government is considering an ad valorem tax or subsidy on good 2. Can the deadweight loss (or benefit) from the tax on good 2 be calculated using the supply and demand curves for good 2? Explain. (4 points)

iv. Find the optimal second best tax (or subsidy) on good 2 (as a function of \( \tau_1 \) ) and explain the role played by \( \gamma \) in determining this second best tax. (6 points)

(b) An individual whose utility depends on consumption of two goods has the following Hicksian demand for good 1:

\[
x_1^h = U^2 \left( \frac{p_2}{p_1} \right)^{1/2}
\]

Currently \( (p_1, p_2) = (9, 9) \) and the person’s Hicksian demand for good 1 is 4. He has the opportunity to shop at a store at which \( p_1 = 4 \); however, if he shops there, he must buy at least 12 units of good 1 (he may not resell them). **What is the maximum amount the individual would pay to shop at the store?** Show how you got your answer. (11 points)
5. Answer all parts

a) Sara’s utility depends upon her leisure \((l)\) and her consumption \((c)\). She is endowed with \(T\) units of time, to be divided between work \((L)\) and leisure \((l)\). The price of consumption, \(p = 1\), so given her wage, her budget constraint, time constraint and preferences, are given by:

\[ wL - c \geq 0; \quad l = (T - l); \quad U = A \cdot l + \left(4/3\right)c^{3/4}; \quad T = 24 \]

Sara can stay in Ames, and receive the wage rate \(w_0\) with certainty, or she can move to New York city. If she goes to New York, she is not sure what her wage will be, but she has the following beliefs about her job prospects:

With probability \(1/2\) \(w = (w_0/2)\);
With probability \(1/2\) \(w = (3w_0/2)\);

Suppose that Sara cannot choose to return to Ames if she winds up with the lower-paying job in New York.

i. Assume Sara must work an 8 hour day, regardless of which job she takes. (there is no labor-leisure choice). Will Sara move to New York? (i.e., will her expected utility be higher if she stays in Ames or moves to New York?). Justify your answer. \(6\) points

ii. For this part assume that Sara can choose how much to work after she finds out her true wage. Assuming an interior solution \((T > l > 0)\), will Sara move to New York? Justify your answer and, if it differs from that in part (i), explain why. \(9\) points

b) Consider a competitive firm that produces an output, \(y\), using four inputs, with the following production function:

\[ y = F(\phi(x_1, x_2), \theta(x_3, x_4)) \]

Functions \(\phi(\cdot)\) and \(\theta(\cdot)\) are concave and homogeneous of degree one, and \(F(\cdot)\) is concave and homogenous of degree one-half. Output price is \(p_y\) and input prices are \((w_1, w_2, w_3, w_4)\).

i. Suppose \(w_1\) increases. How will this affect the conditional factor demands for each input? Be as specific as possible. \(5\) points

ii. Assuming the firm is a competitive profit-maximizer, what is the firm’s supply elasticity (i.e., what is \((dy^*/dp)(p/y^*)\))? Explain your answer. \(5\) points

iii. Assume \(\phi(x_1, x_2) = x_1^{1/2}x_2^{1/2}\) and \(\theta(x_3, x_4) = Min(x_3, x_4)\). Suppose initial output and factor prices change from \(\left\{p_y = 8, (w_1, w_2, w_3, w_4) = (4, 2, 2, 4)\right\}\) to \(\left\{p_y = 16, (w_1, w_2, w_3, w_4) = (6, 3, 3, 9)\right\}\). How will this change in prices affect the firm’s profits, its profit maximizing output supply, and its factor demands? Be as specific as possible and explain your answer. \(8\) points