Economics 601, Microeconomic Analysis I
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Midterm 2
Answer Any Three Questions

1. Answer all parts.

(a) Consider a utility maximizing consumer with utility function: 
\[ U(x_1, x_2, x_3) = \phi(x_1, G(x_2, x_3)) \]
where \( \phi(x_1, G) \) is a strictly quasi-concave function and 
\( G(x_2, x_3) = \left(\frac{3}{2}x_2^{2/3}\right) \left(x_3^{1/3}\right) \). The consumer has income \( \omega \) and pays prices \((p_1, p_2, p_3)\).

i. The individual has the choice of shopping in one of two stores (she cannot shop in both). In store A prices are 
\( (p_1^A, p_2^A, p_3^A) = (1, 4, 2) \) whereas in store B prices are 
\( (p_1^B, p_2^B, p_3^B) = (1, 2, 6) \). In which store will the person shop? Prove your answer. (8 points)

ii. Consider the Slutzky matrix whose typical element is 
\( \frac{\partial \hat{x}_i^h}{\partial p_j} \); \( i, j = 1, 2, 3, \) where \( \hat{x}_i^h \) is the Hicksian demand function for good \( i \). Draw a 3x3 matrix and indicate, where possible, the sign of each element. Be as specific as possible. (5 points)

iii. Consider the Marshallian demands for goods 2 and 3. What determines the sign of 
\( \frac{\partial \hat{x}_2^*}{\partial p_1} \) and what can you conclude about the sign of 
\( \frac{\partial \hat{x}_2^*}{\partial p_1} \) and the sign of 
\( \frac{\partial \hat{x}_3^*}{\partial p_1} \)? Relate your answer to the price elasticity of demand for good 1, and be as specific as possible. (6 points)

(b) Consider a utility maximizing consumer who consumes 3 goods. Suppose the person’s Hicksian demand for good 1 is: 
\[ x_1^h = \frac{U^2}{\left[1 + p_1(p_2)^{2/3}(p_3)^{-1/3}\right]^2} \].
Originally, the person has income \( \omega = 48 \), faces the price vector \((p_1, p_2, p_3) = (4, 2, 2)\), and consumes four units of good 1.

i. Suppose the price of good 1 increases to 8, so the new price vector is \((p_1, p_2, p_3) = (8, 2, 2)\). What is the compensating variation to this price change? (8 points)

ii. Suppose instead the price vector changes to \((p_1, p_2, p_3) = (4, 1, 1)\). Compared to the initial situation \((\omega = 48, \quad (p_1, p_2, p_3) = (4, 2, 2))\) what is the compensating variation for this price change? Explain how you got your answer. (6 points)
2. Answer All Parts.

(a) Consider an individual whose utility depends on her consumption of two goods \((x_1, x_2)\) and leisure \((l)\). She is endowed with \(T = 24\) units of time, which is divided between leisure and work \((L)\). The prices of the goods, and the wage rate, are \((p_1, p_2, W)\), all measured in dollars, and she has debts (from college) of \(D\) (also measured in dollars). Her net full income is \(Y^f = WT - D\). Her preferences are given by:

\[
U = \left\{ l^\rho + \left( 4x_1 x_2 \right)^{\rho/2} \right\}^{\frac{1}{\rho}}, \quad \rho \equiv \left( \frac{\sigma - 1}{\sigma} \right), \quad \sigma > 0, \quad \sigma \neq 1.
\]

i. Find her utility maximizing consumption demands and labor supply.

ii. Suppose the government taxes her earned income, so that after tax income – which is used to buy goods \((x_1, x_2)\) and repay her debt, is \(WL(1 - T)\). How does a change in the tax rate affect how much she works? Relate your answer to \(\sigma\). What role does her debt play in the decision? Explain.

iii. The individual can live in Ames, where wages and prices are: \((W, p_1, p_2) = (20, 4, 1)\), or move to New York city, where wages and prices are: \((W, p_1, p_2) = (40, 4, 4)\). Will she move and what role does her debt play in the decision?

(b) Consider a firm which produces an output, \(q\), using two inputs \((x_1, x_2)\). Let \(p\) denote output price, and \(w_1, w_2\) be input prices. Further, let \(\hat{x}_i(q, w_1, w_2)\) denote the conditional factor demands, \(x_i^*(p, w_1, w_2)\) denote profit maximizing factor demands and \(q^*(p, w_1, w_2)\) denote profit maximizing output supply. Finally, suppose you are told that input 2 is locally inferior.

i. What are the signs of \((\partial \hat{x}_i / \partial w_2)\), and \((\partial x_i^* / \partial w_2)\), \(i = 1, 2\)? What is the sign of \((\partial q^* / \partial w_2)\)? Prove your answer.

ii. Compare the magnitudes of \((\partial \hat{x}_i / \partial w_2)\) and \((\partial x_i^* / \partial w_2)\), \(i = 1, 2\).
3. Answer All parts

(a) Consider a firm with production function: \( q = f(x_1, x_2) + g(x_2, x_3) \), where \( f(\cdot) \) and \( g(\cdot) \) are strictly increasing, and strictly concave, in their arguments. Let \( p \) denote output price, and \( \mathbf{w} = (w_1, w_2, w_3) \) denote input prices. Suppose inputs one and two are variable in both the short run and long run, whereas input three is variable only in the long run (its short run value is fixed at \( x_3 = \bar{x}_3 \)).

Assuming all inputs are normal, contrast the short run and long run impact of an increase in output price on profit-maximizing output and the profit-maximizing factor demands for inputs 1 and 2. Be as specific as possible and relate your answer to the cross partial derivatives of the production function. (8 points)

(b) A firm produces an output, \( y \), using three inputs. Let output price be \( p \), and input prices \( (w_1, w_2, w_3) \).

The firm’s output supply curve and profit-maximizing demand for input one are given by:

\[
\begin{align*}
y^* &= \frac{3p^2}{w_1 w_2^{1/2} w_3^{1/2}}; \\
x_1^* &= \frac{p^3}{w_1^{1/2} w_2^{1/2} w_3^{1/2}}
\end{align*}
\]

Initially, the price vector is \( (p, w_1, w_2, w_3) = (2, 1, 1, 1) \) and profits are \( \pi^* = 8 \).

i. Calculate the firm’s profits if the price vector changes to \( (p, w_1, w_2, w_3) = (4, 4, 1, 1) \). (8 points)

ii. Give an upper bound on profits if prices are \( (p, w_1, w_2, w_3) = \left(5, \frac{9}{2}, \frac{3}{2}, \frac{3}{2}\right) \).

HINT: \( \left(5, \frac{9}{2}, \frac{3}{2}, \frac{3}{2}\right) = \frac{1}{2} (2, 1, 1, 1) + \frac{1}{2} (8, 8, 2, 2) \) (4 points)

(c) A firm has production function \( q = \left[9x_1 + x_2^2\right]^{1/4} \).

Let \( p \) denote output price and \( (w_1, w_2) \) denote input prices.

i. Find the firm’s cost function \{HINT – is this function quasi-concave?\}. (7 points)

ii. Let \( (w_1, w_2) = (1, 1) \). Find the firm’s profit-maximizing supply curve. \{HINT – is marginal cost continuous?\} (6 points)
4. Answer All Parts

(a) Consider a firm which produces an output \( q \), using inputs \( (x_1, x_2) \) with the following production function:

\[
q = f(x_1, x_2) = \left[ x_1^\alpha + x_2^\alpha \right]^{1/\alpha}
\]

Let input prices be \( w_1, w_2 \) and output price be \( p \).

i. What restrictions on \( \alpha, \lambda \) must hold in order for there to be a solution to the cost minimization problem? (i.e., \( \text{Min}_{x_1, x_2} \{w_1 x_1 + w_2 x_2\} \) such that \( f(x_1, x_2) \geq q_0 \)). (2 points)

ii. What restrictions on \( \alpha, \lambda \) must hold in order for there to be a solution to the profit maximization problem? (i.e., \( \text{Max}_{x_1, x_2} \{pf(x_1, x_2) - w_1 x_1 - w_2 x_2\} \)). (2 points)

iii. Let \( \alpha = (-1/2), \lambda = (1/2) \). Derive the firm’s cost function and maximum profit function. (12 points)

(b) Suppose a firm has the following profit function: \( \pi(p, w_1, w_2) = \left( \frac{p^2}{w_2} \right) \left( \frac{2p}{w_1} + 1 \right)^2 \), where \( p \) is output price and \( (w_1, w_2) \) are input prices.

i. Find the firm’s output supply and factor demand curves. (7 points)

ii. Find the firm’s production function. (10 points)