Economics 601 GianCarlo Moschini
Microeconomic Analysis I Fall 2008

Problem Set No. 1 Due by: Friday, August 29

1.1. Graph the sets A and B below and decide whether or not they are convex.

$$A = \left\{ (x, y) : y = x^2 \right\}$$
$$B = \left\{ (x, y) : y \le \ln x \right\}$$

1.2. Decide whether the following functions are quasi-concave.

$$f(x) = 5 + 2x$$
$$f(x_1, x_2) = x_1 e^{x_2}$$

1.3. Decide whether the following functions are concave or convex.
[Hint: recall "useful properties" of concave and convex functions.]

$$f(x_1, x_2, x_3) = ax_1^2 + bx_2^2 + cx_3^2 \qquad (a, b, c \ge 0)$$

$$g(x_1, x_2, x_3) = e^{ax_1^2 + bx_2^2 + cx_3^2} \qquad (a, b, c \ge 0)$$

$$h(x_1, ..., x_n) = \log(a_1 x_1 + ... + a_n x_n), \qquad (a_1 x_1 + ... + a_n x_n > 0)$$

1.4. Consider the function $f : \mathbb{R}^2_{++} \to \mathbb{R}$ specified as $f(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$.
a. Derive restrictions on α and β which ensure that f is concave
b. Derive restrictions on α and β which ensure that f is quasiconcave
c. Derive restrictions on α and β which ensure that f is convex

1.5. Let $f : [-2, 2] \to \mathbb{R}$ be given by $f(x) = 2x^3 - 3x^2$.
Use first and second order conditions to identify local minimum and maximum points, and argue why these local optima cannot be global optima.

1.6. For the following three versions of the function $f : S \to \mathbb{R}$ illustrate why Weierstrass theorem does not apply:

(i) $S = \mathbb{R}$ and $f(x) = x^3$

(ii) $S = (0,1)$ and $f(x) = x$

(iii) $S = [-1,1]$ and $f(x) = \begin{cases} 0 & if \quad x = -1 \; or \; x = 1 \\ x & otherwise \end{cases}$