Economics 601 GianCarlo Moschini
Microeconomic Analysis I Fall 2008

Problem Set No. 11 Due by: Friday, November 21

1. The Expected Utility (EU) property is cardinal in the sense that it is not preserved under an arbitrary monotonic transformation. On the other hand, this property is preserved under an increasing linear transformation. That is, suppose that $U(L)$ represents and agent's preferences over lotteries and possesses the EU property. Define $\tilde{U}(L) \equiv \kappa + \gamma U(L)$, where $\gamma > 0$. Then, prove that:

   (a) $\tilde{U}(L)$ provides the same ranking over lotteries as $U(L)$ does.
   (b) $\tilde{U}(L)$ has the expected utility property.

2. (Allais Paradox) Considers the following four lotteries:

$$L_1 = \begin{cases} \text{win prize } A \text{ with probability } & \alpha \\ \text{win prize } B \text{ with probability } & \beta \\ \text{win prize } C \text{ with probability } & 1-\alpha-\beta \end{cases} \qquad L_2 = \text{ win prize } B \text{ for sure}$$

$$L_3 = \begin{cases} \text{win prize } B \text{ with probability } & 1-\beta \\ \text{win prize } C \text{ with probability } & \beta \end{cases} \qquad L_4 = \begin{cases} \text{win prize } A \text{ with probability } & \alpha \\ \text{win prize } C \text{ with probability } & 1-\alpha \end{cases}$$

   Suppose that an agent's preferences are such that $L_2 \succ L_1$ and $L_4 \succ L_3$. Show that these preferences violate the Expected Utility Theorem.

   Note that the above formulation corresponds to the experiment carried out in class, where we had $A = \$\ 5$ mil., $B = \$\ 1$ mil., and $C = \$\ 0$, and the four lotteries were: $L_1 = (0.10, 0.89, 0.01)$, $L_2 = (0, 1, 0)$, $L_3 = (0, 0.11, 0.89)$, and $L_4 = (0.10, 0, 0.90)$. Using these values, illustrate the violation of the EU property by representing the four lotteries on the "probability triangle."

3. An agent has a Bernoulli utility function given by $u(w) = \ln(w)$. She is offered the opportunity to bet on the flip of a coin that has a probability π of coming up heads (and probability $1-\pi$ of giving tails). If she bets \$$x$ the payoff will be $w + x$ if heads comes up and $w - x$ if tails comes up. Solve for the optimal size of the bet x as a function of π. What is his optimal choice of x when $\pi = 1/2$?

4. Consider two risk-averse individuals with utility functions $u_1(x)$ and $u_2(x)$, and let $c(F, u_1)$ and $c(F, u_2)$ denote the certainty equivalent functions for these two agents. Show that, if agent two is more risk averse than agent one in the sense that $u_2(x) = \psi(u_1(x))$, where $\psi(.)$ is a concave function, then $c(F, u_2) \le c(F, u_1)$ [for any $F(.)$].

5. Consider the insurance problem analyzed in class. Show that a strictly risk averse individual purchases less than full insurance if $q > \pi$ (i.e., when the insurance premium is not actuarially fair).