Economics 601 GianCarlo Moschini
Microeconomic Analysis I Fall 2008

Problem Set No. 12 Due by: Friday, December 5

1. Problem 6.C.15 in MWG textbook (page 212).

2. Consider the expected utility maximization problem of the risk-averse firm discussed in class:

$$\max_{q} \int u\left(w_0 + (\mu + \gamma z)q - C(q)\right) dF(z)$$

where q is output, w_0 denotes initial wealth, μ is expected price, $\gamma > 0$ is a parameter controlling the "dispersion" of the price distribution, $u(w)$ is the strictly concave Bernoulli utility function, $C(q)$ is the cost function, and $\tilde{z}$ is a zero-mean random variable with given distribution function $F(z)$.
Let q^* denote the optimal choice of the risk-averse firm. Now consider a change in the dispersion of the random price, i.e., a change in the parameter γ. Note that an increase in γ implements (a special case of) what is known as a "mean preserving spread" of the price distribution. Show that, as the price distribution becomes riskier in this sense, the production level decreases (that is, $\partial q^*/\partial \gamma < 0$) if risk preferences satisfy DARA or CARA (i.e., NIARA).

3. An individual with a monotonically increasing and strictly concave Bernoulli utility function $u(w)$ is evaluating a risky prospect $\tilde{w}$ that is normally distributed with mean μ and variance σ^2. Specifically, $\tilde{w} \equiv \mu + \sigma\tilde{\varepsilon}$ and $\tilde{\varepsilon} \sim N(0,1)$. Then, clearly, the expected utility of this lottery is a function of the mean and variance parameters, i.e., $E\left[u(\tilde{w})\right] \equiv V(\mu, \sigma^2)$. [Note: here we are <u>not</u> assuming CARA].

   (a) Show that $V(\mu, \sigma^2)$ is increasing in μ.
   (b) Show that $V(\mu, \sigma^2)$ is decreasing in σ^2.

4. Consider the simple portfolio allocation problem analyzed in class, where $\tilde{w} = w_0 + (\tilde{z} - 1)\alpha$ and α is the amount of wealth allocated to the risky asset. Suppose that the risky asset's return is normally distributed, specifically $z \sim N(\mu, \sigma^2)$. And, suppose that the Bernoulli utility function of the decision maker is of the negative exponential type (i.e., CARA), that is $u(w) = -e^{-\lambda w}$, $\lambda > 0$. Given the result discussed in Handout # 11, formulate the problem in the mean-variance framework and derive explicitly the demand function for the risky asset α^*. Show how α^* is affected by the parameters of the problem (i.e., μ, σ^2 and λ).