Economics 601 GianCarlo Moschini
Microeconomic Analysis I Fall 2008

Problem Set No. 13 Due by: Friday, December 12

1. Problem 10.C.3 in MWG textbook (p. 344).

2. Problem 10.C.10 in MWG textbook (p. 346).

3. Consider the market for good x. The supply side consists of an industry with J identical producers, and the demand side consists of N consumers with quasi-linear (but not identical) preferences.

   (a) If the industry can be represented by a "representative firm" with aggregate cost function $C(Q) = Q^2/2$, what is the form of the individual firms' cost function?

   (b) Suppose that individual utility functions can be written as:

   $$u_i(x_i, y_i) = y_i + \beta_i \log(x_i) \qquad \beta_i > 0, \forall i \qquad i = 1, 2, ..., N$$

   What is the form of the utility function for the "representative consumer"?

   (c) Now use the cost function of the "representative firm" in part (a) and the utility function of the "representative consumer" derived in part (b).
   - Solve for the competitive equilibrium and show that this competitive equilibrium maximizes social welfare.
   - Suppose that an "*ad-valorem*" tax $\tau > 0$ is imposed on good x so that, if p is the price received by the representative firm, the consumer pays $(1+\tau)p$. Solve for the tax-distorted competitive equilibrium, and compute the deadweight loss due to this tax.

4. Let $c(w, q)$ denote the cost function of a competitive firm, where q is output and w is the vector of input prices. Assume that it takes the following form:

   $$c(w, q) = \begin{cases} 4w_1 + q^2\sqrt{w_1 w_2} & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

   (a) Find the firm's profit-maximizing supply function. Now suppose that $w_1 = w_2 = 1$, and let p represent output price. What is the firm's optimal output if $p = 6$? What about when $p = 3$?

   (b) Assume that there is free entry in this competitive market, and that input prices are $w_1 = w_2 = 1$. What is the long-run supply correspondence for this industry?

   (c) The output of this industry is demanded by 1,000 consumers, each with indirect utility function $V_i = \omega_i - p + p^2/10$, where i indexes consumers and ω denotes income (measured in units of a numeraire good). Input prices are still assumed fixed at $w_1 = w_2 = 1$. Determine the long-run equilibrium (including the long-run number of firms) in this market.

5. Consider a competitive industry in long run equilibrium. All firms are identical, each with cost function $C(w,q)$, where q denotes the output of one firm, and w is the vector of (exogenously given) input prices. This cost function displays a U-shaped average cost and a strictly increasing marginal cost. The (downward sloping) market demand for this industry is written as $x(p)$, where p denotes the price for the industry output. The industry long-run equilibrium is characterized by the triple $\{p^*,q^*,J^*\}$, where J denotes the number of firms. [Strictly speaking J is an integer, but you can ignore that and treat J as a real number].

   (a) Write down the system of equations that define the long-run equilibrium. Briefly discuss the rationale behind each of the equations.

   (b) Use comparative statics on the system of equations derived in (a) to determine the impact, on the long run equilibrium, of an increase in an input price w_k. Specifically, determine the signs of $\partial p^*/\partial w_k$, $\partial q^*/\partial w_k$, and $\partial J^*/\partial w_k$ under the assumption that w_k is the price of an *inferior input.* [Recall that an input is said to be inferior if the cost-minimizing input demand is negatively related to output, i.e., $\partial h_k(w,q)/\partial q \le 0$].