

Problem Set No. 4

Due by: Friday, October 3

4.1 Stone-Geary utility function

Assume that, for the 2-good case, the utility function has the so-called *Stone-Geary* form:

$$u(x_1, x_2) = (x_1 - \gamma_1)^\alpha (x_2 - \gamma_2)^\beta$$

- Explain why we should have $\alpha > 0$ and $\beta > 0$, and, without loss of generality, why it is possible to put $\alpha + \beta = 1$. (For the remainder of the problem, maintain these conditions).
- Standard theory does not restrict the sign of parameters γ_1 and γ_2 (why?). Here, however, assume that $\gamma_i > 0$, $i = 1, 2$. Given that, define an appropriate consumption set for these preferences and provide a graphical illustration.

For the remainder of this problem, assume that an interior solution applies.

- Set up and solve UMP, and derive the indirect utility function. Verify that the indirect utility function satisfies the appropriate homogeneity property.
- Set up and solve EMP, and derive the expenditure function. Verify that the expenditure function satisfies the appropriate curvature property.
- Verify that the indirect utility function can be alternatively obtained by inverting the expenditure function, and, correspondingly, that the indirect utility function can be obtained by inverting the expenditure function. Briefly explain why this procedure is legitimate.

4.2 Substitution matrix (MWG 3.G.14)

Let $S(p, w)$ denote the $n \times n$ matrix of substitution effects that can be calculated from Marshallian demands via the Slutsky equation. Specifically, the ij^{th} element of the matrix $S(p, w)$ is:

$$S_{ij} \equiv \frac{\partial x_i(p, w)}{\partial p_j} - x_j(p, w) \frac{\partial x_i(p, w)}{\partial w}$$

Suppose that, when $n = 3$, we have $p_1 = 1$, $p_2 = 2$, and $p_3 = 6$ and the matrix $S(p, w)$ is:

$$\begin{bmatrix} -10 & ? & ? \\ ? & -4 & ? \\ 3 & ? & ? \end{bmatrix}$$

where “?” indicates missing numbers.

- a. Compute the missing numbers. Briefly explain the procedure you use in each step.
- b. Does the resulting matrix satisfy all the properties of a substitution matrix? Explain.

4.3 Slutsky compensation

Consider the standard utility maximization problem of consumer theory, and let $x_i(p, w)$ and $h_i(p, u)$ denote the usual Marshallian and Hicksian demand functions, respectively, where $p \in \mathbb{R}_{++}^n$ is the price vector, $w \in \mathbb{R}_{++}$ is income, and $u \in \mathbb{R}$ is a utility level. Hicksian demands are commonly referred to as “compensated demand functions” because they can be interpreted as arising when the consumer is “compensated” for the change in real income due to a price change (with an income adjustment that is just enough to allow the consumer to achieve, and the new prices, the same level of utility possible before the price change). An alternative compensation criterion, the so-called Slutsky compensation, provides the consumer with enough income to actually buy the bundle that was optimal before the price change.

- (a) For the case on $n = 2$, illustrate the Hicks-compensation and the Slutsky-compensation criteria in a diagram. For any finite price change, would the consumer be indifferent between the two compensation criteria or would she prefer one of them?
- (b) Now define the “Slutsky-compensated” demand functions $s_i : \mathbb{R}_{++}^n \times \mathbb{R}_+^n \rightarrow \mathbb{R}_+$ as:

$$s_i(p, x^0) \equiv x_i(p, p \cdot x^0) \quad , \quad i = 1, 2, \dots, n$$

Prove that the slopes of Hicksian demand functions and Slutsky-compensated demand functions, when evaluated at the point (p^0, x^0, u^0) , satisfy:

$$\frac{\partial s_i(p^0, x^0)}{\partial p_j} = \frac{\partial h_i(p^0, u^0)}{\partial p_j} \quad , \quad \forall i, j = 1, 2, \dots, n$$

where $x_i^0 \equiv x_i(p^0, w^0)$, $\forall i$, and $u^0 \equiv u(x^0)$. That is, the slopes of Hicksian and Slutsky-compensated demand functions are locally the same, despite the conclusion of part (a) for finite price changes. (Incidentally, the foregoing explains why the matrix of substitution effects derived from Hicksian demand functions is typically referred to as the “Slutsky matrix”).